Solid Mechanics - 202041

Mr. K. B. Bansode

Mechanics

- The solid mechanics as a subject may be defined as a branch of applied mechanics that deals with behaviors of solid bodies subjected to various types of loadings.

1. Mechanics of rigid bodies
2. Mechanics of deformable solids/Mechanics of solids
3. Fluid Mechanics

Unit I Simple stresses and strains

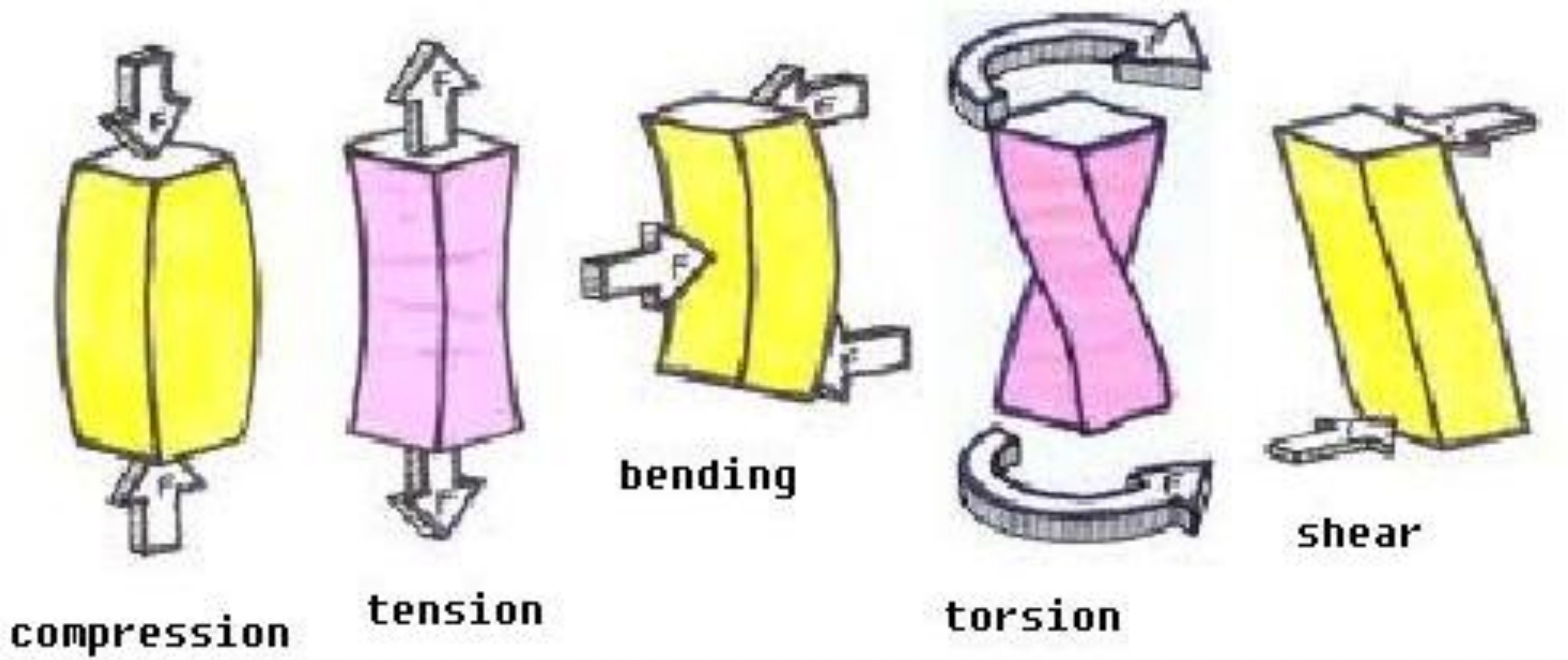
- CO1. DEFINE various types of stresses and strain developed on determinate and indeterminate members.

Simple Stress & Strain: Introduction to types of loads (Static, Dynamic & Impact Loading) and various types of stresses with applications, Hooke's law, Poisson's ratio, Modulus of Elasticity, Modulus of Rigidity, Bulk Modulus. Interrelation between elastic constants, Stress-strain diagram for ductile and brittle materials, factor of safety, Stresses and strains in determinate and indeterminate beam, homogeneous and composite bars under concentrated loads and self-weight, Thermal stresses in plain and composite members

Static / Dynamic

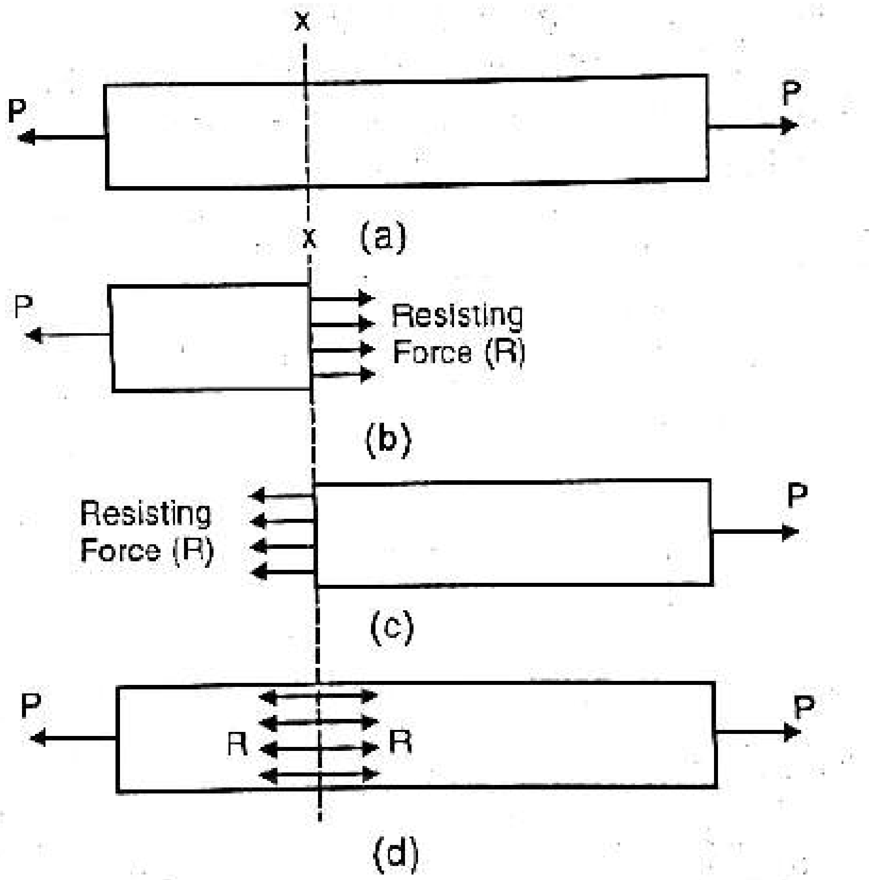
- Static: remains constant over the period
  - External force will form equilibrium internal restoring force
  - Deformation or Stresses depend on external force and stiffness
  - A good example of a static load is the weight of a building acting on the ground. Another example is a car parked at a car park.
- Dynamic: varies with respect to time
  - External force will form equilibrium Inertia force, damping force and internal restoring force
  - Deformation or Stresses depend on external force and stiffness, mass and forcing frequency
  - A good example of a dynamic load is the weight of a moving car on the road.

Types of Load and effect of load on Solid



Internal Resistance

- Whenever a body is subjected to an external force, it tends to undergo deformation (i.e. change in shape or dimensions).
- Due to cohesion between the molecules, the body resists deformation. This internal resistance is equal and opposite to the applied force.



Stress

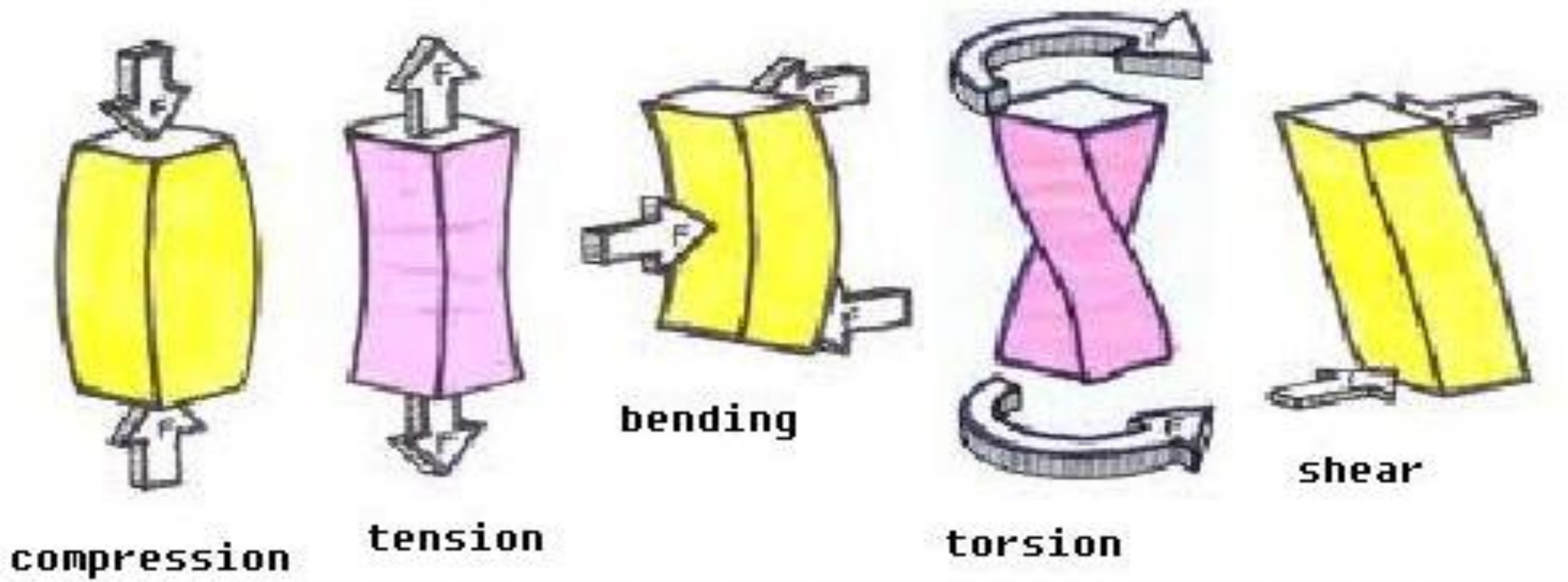
- The internal resistance per unit area of cross section offered by a body against the deformation is called stress.
- Mathematically stress may be defined as the force per unit area i.e.

$$\text{stress}, (\sigma) = P/A$$

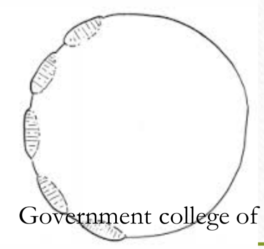
where, P = load or force acting on the body

A = cross-sectional area of the body

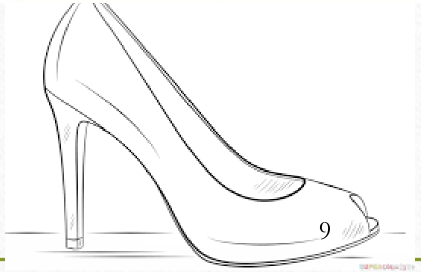
Stress



GAJ and GAJGamini

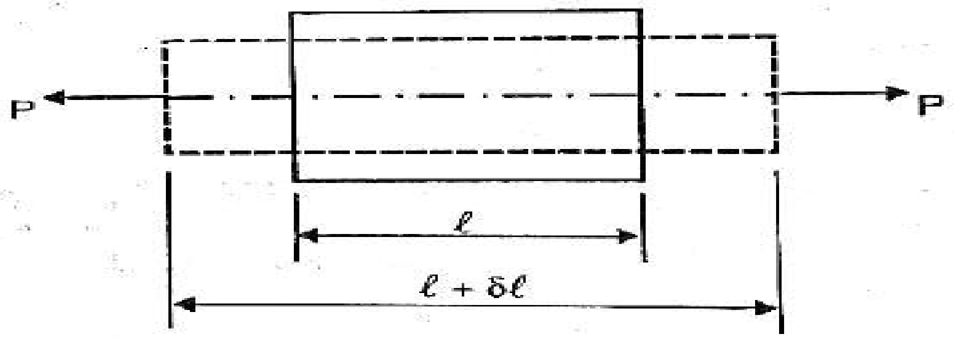


- **GAJ** : 5400/4 = 1350 Kg per leg
- Paw dia 50 CM Area=1962 cm2
- Stress = 1350/1962 = 0.69 Kg/Cm^2
- **Gajgamini** weight 60 kg half 30kg (two legs) half 15kg (only on hills)
- Hill Tip area=0.785cm^2
- Stress=15/0.785=19Kg/Cm^2
- About 20 times Higher than Gaj

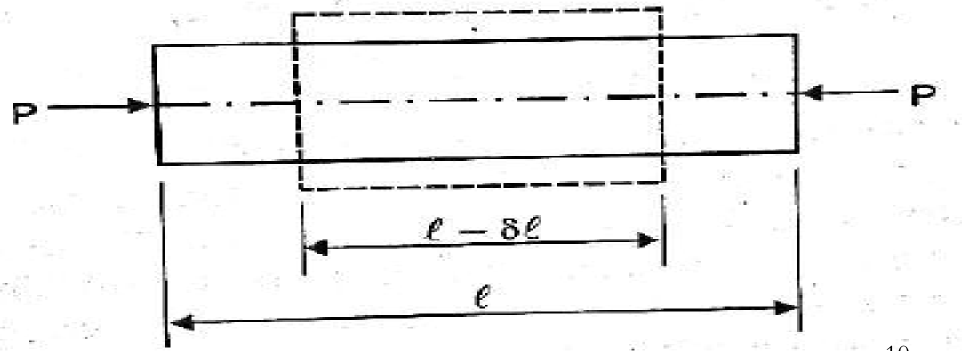


Direct Stress

- Direct stress is that which acts perpendicular to the cross-section of the member.
- Direct stress is also known as normal stress.
- Direct stress is either tensile or compressive in nature according to the nature of loading.



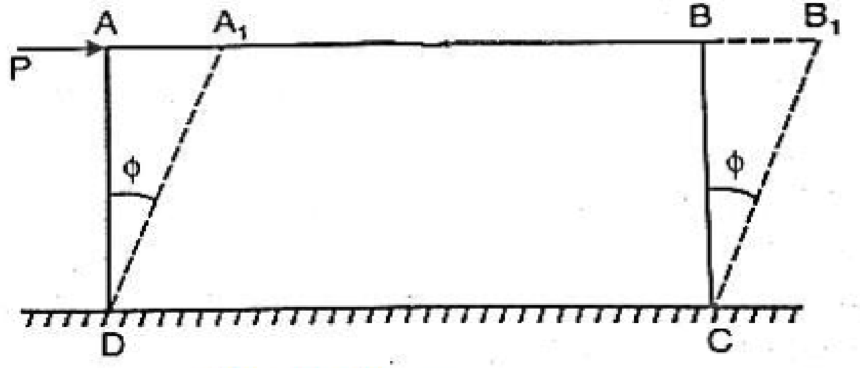
Tensile stress



Compressive stress

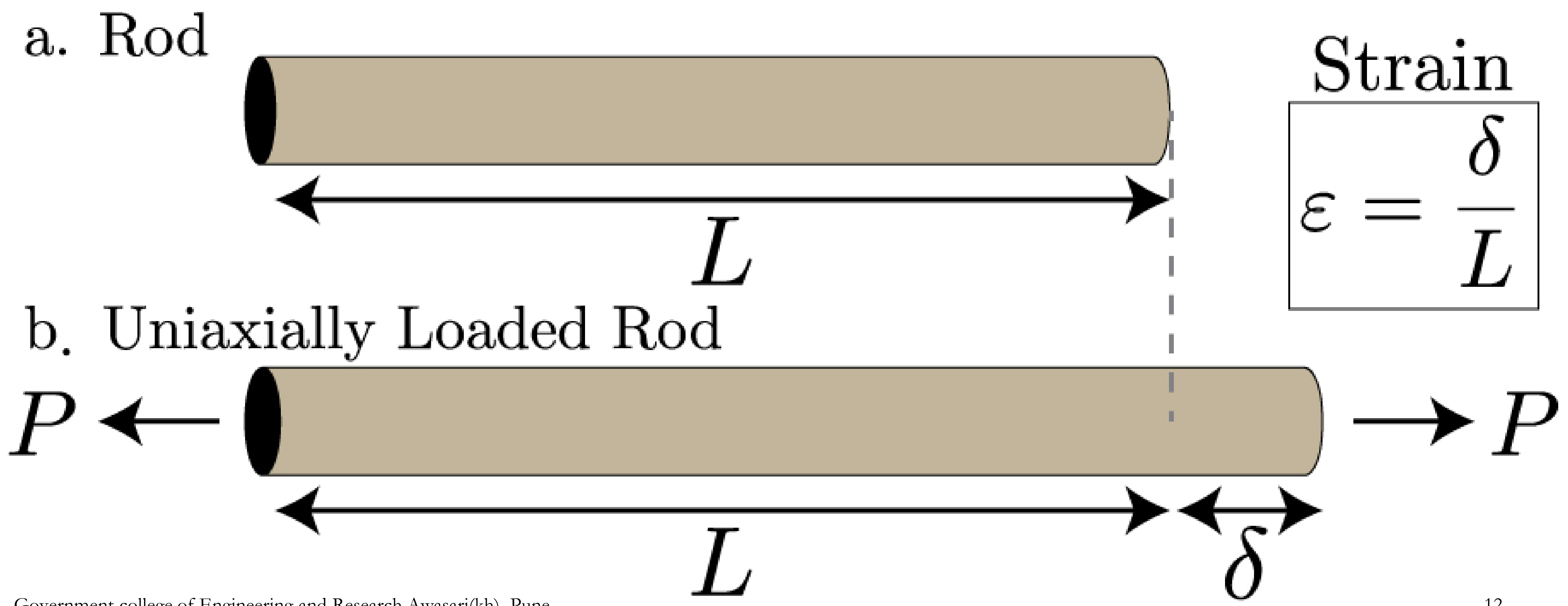
Shear Stress

- When two equal and opposite forces are acting tangentially to the cross-section, the stress induced in the member is known as shear stress.
- Due to these equal and opposite forces, the member tends to shear off across the section.

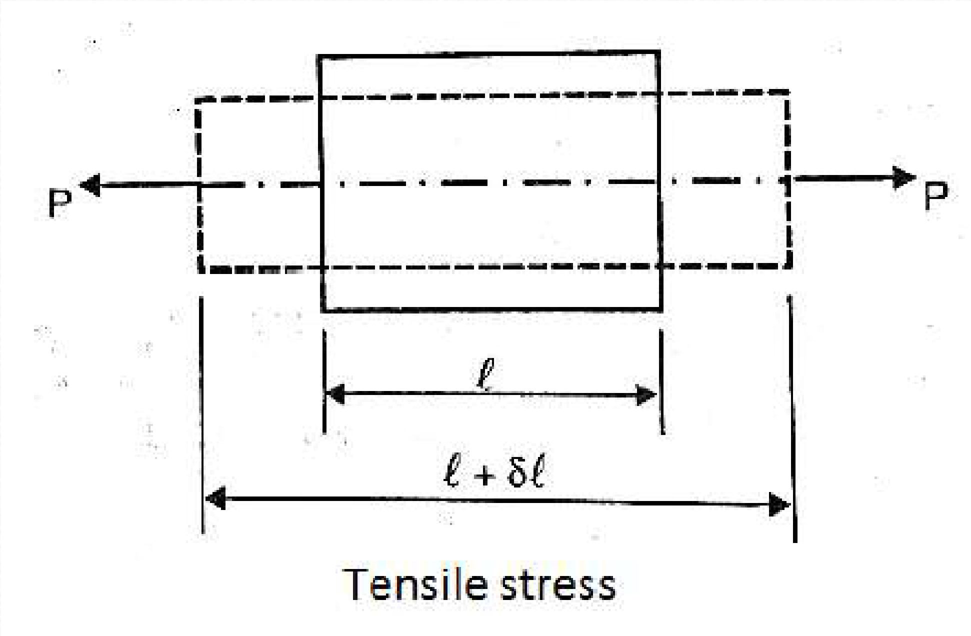


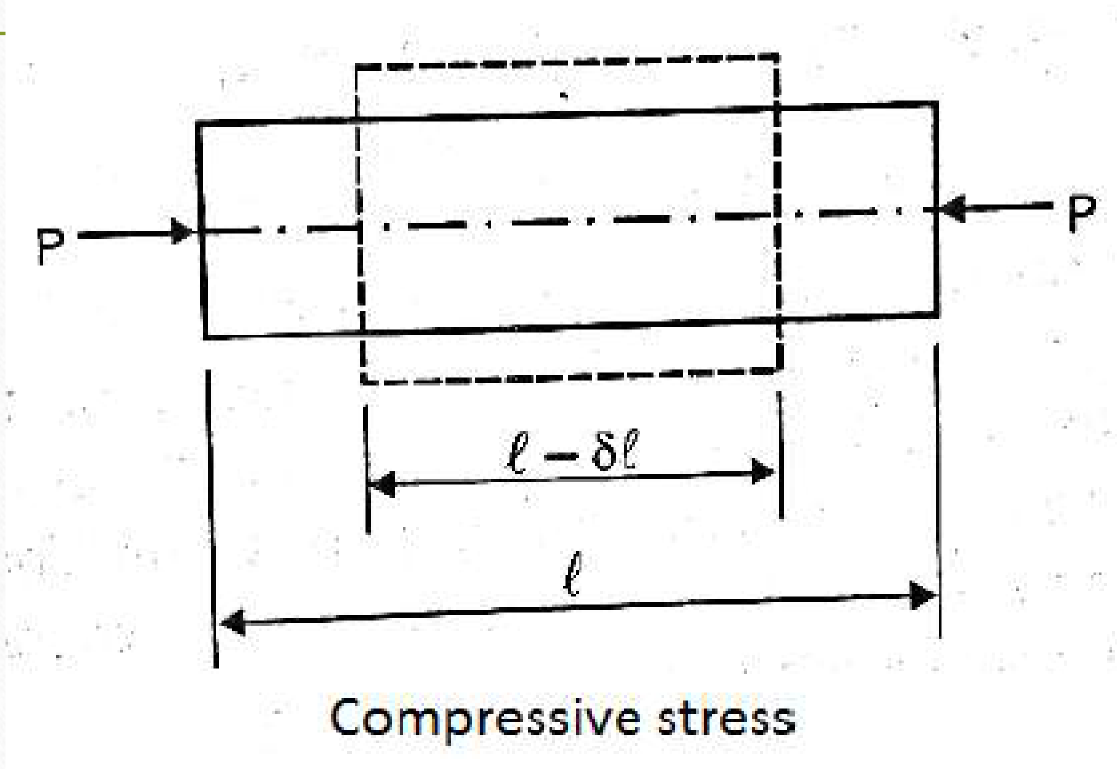
Shear stress

Strain



Direct Strain



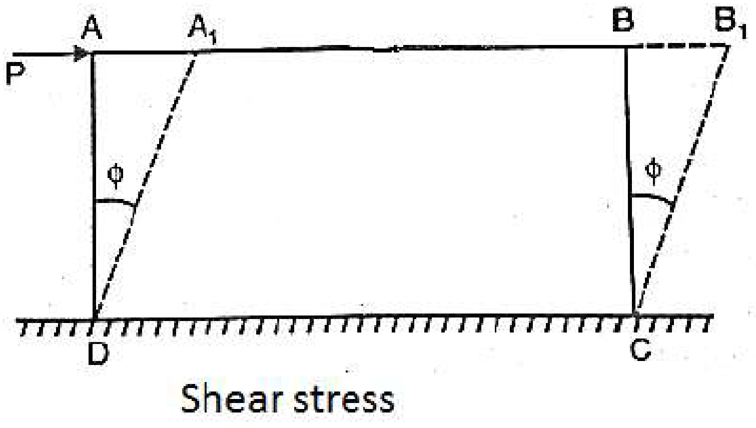


Shear Strain

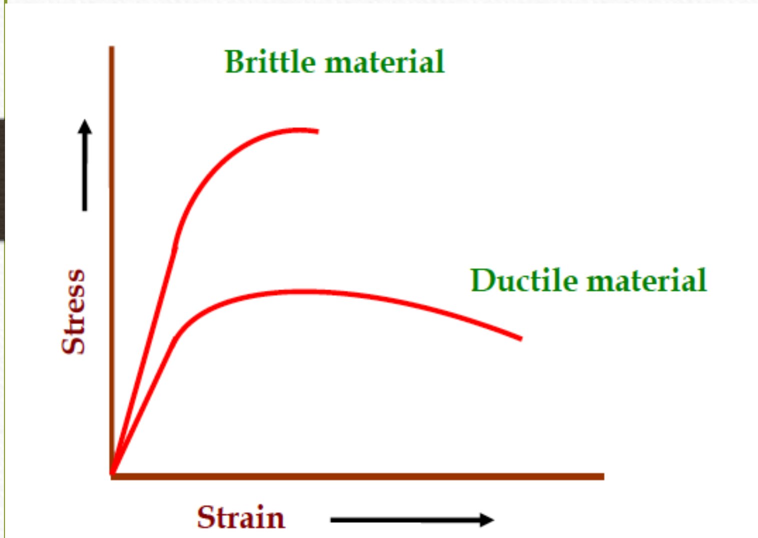
Shear strain- It is a measure of the angle through which a body is distorted under the action of shear forces.

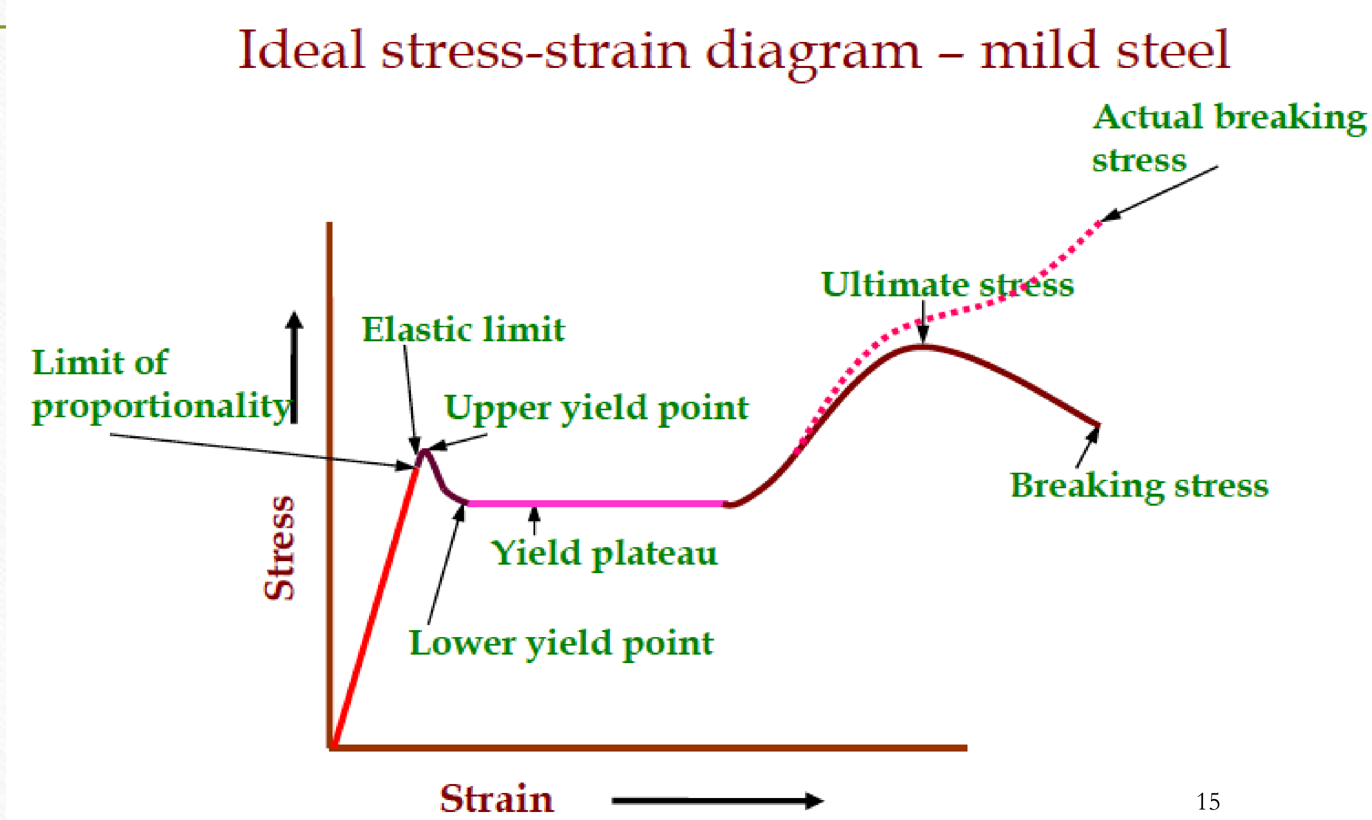
Shear strain, $(e_q) = \tan \phi$ or ϕ (only)

Example- when bolt is turned by a spanner the material of the bolt is in a state of shear strain.



Stress-Strain Diagram



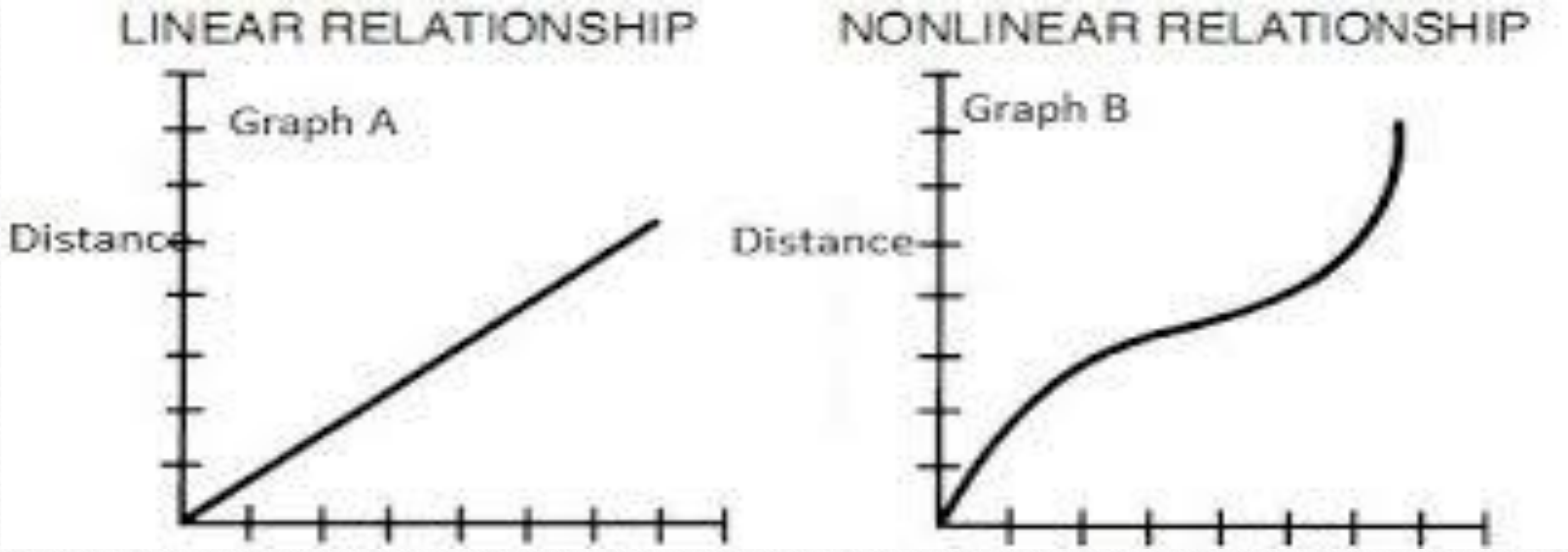


$$\text{Factor of Safety} = \frac{\text{yield stress}}{\text{working stress}}$$

Structural Properties

- Elasticity: Property by virtue of which a deformed body under the action of loads regains its original shape once the loads are removed.
- Homogeneity: Same composition throughout body; Elastic properties are same at every point in the body; Elastic properties need not be same in all directions
- Isotropy: Elastic properties are same in all directions
- Anisotropy: Elastic properties different in various directions

Linear / Non-Linear

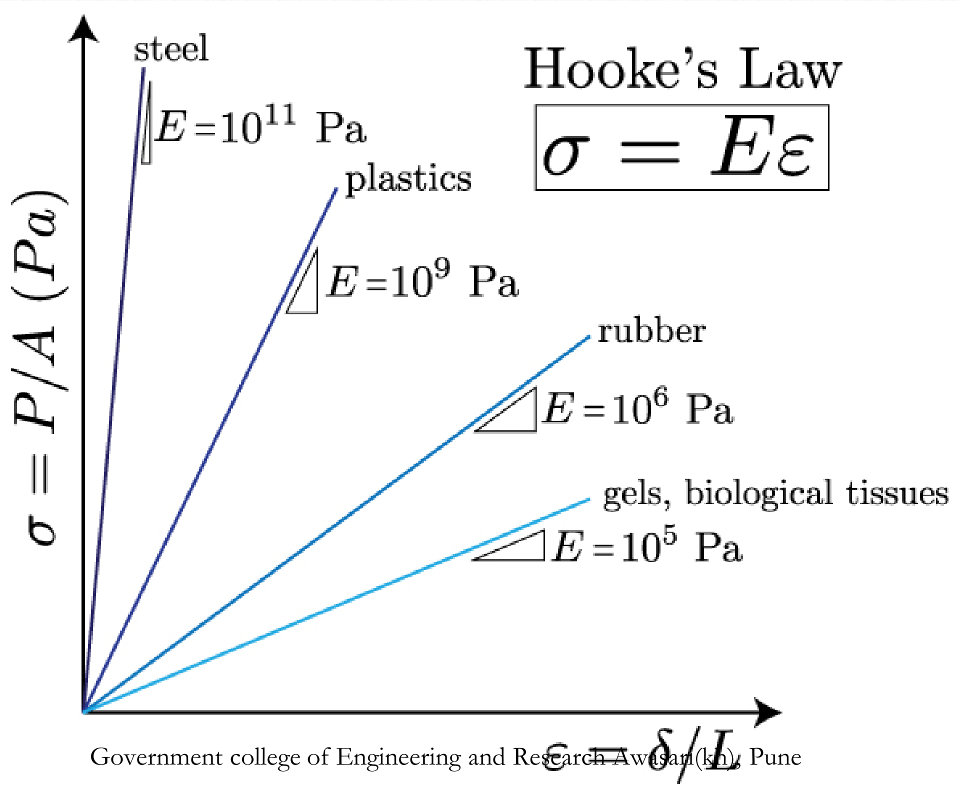


Stiffness / Strength

- Stiffness is resistance to deform

- Strength is Resistance to failure

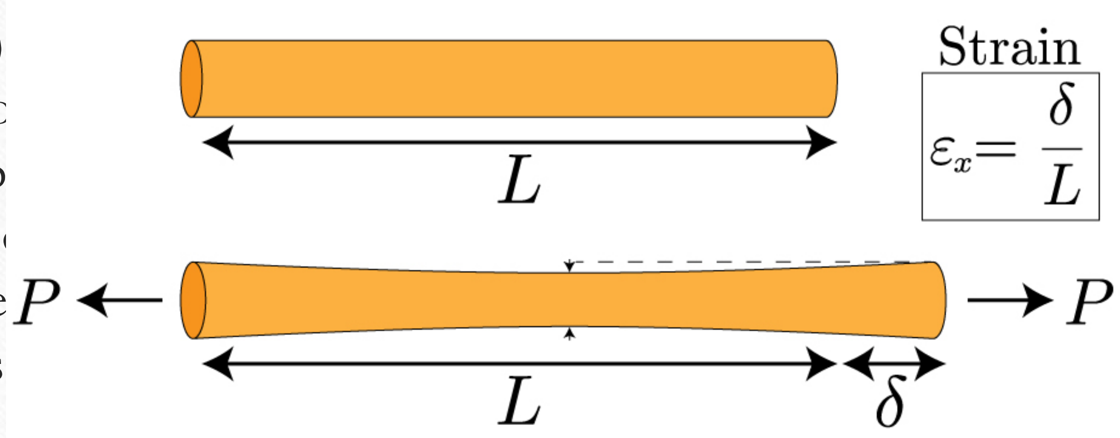
Hooke's Law



- Stress and strain are related by a **constitutive law**,
- The linear, elastic relationship between stress and strain is known as **Hooke's Law**.

Poisson's ratio

- First things first, even just pulling (or pushing) materials in **one direction** actually causes defc in **all three** orthogonal **directions**. Let's go b first illustration of strain. This time, we will ac the fact that pulling on an object **axially** cause compress **laterally** in the transverse directions
- **Poisson's ratio is a material property**.
- For most engineering materials, for example steel or aluminum have a Poisson's ratio around 0.3, and rubbers have a Poisson's ratio around 0.5,



$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

EXAMPLE 6.3. *A steel bar 2 m long, 20 mm wide and 15 mm thick is subjected to a tensile load of 30 kN. Find the increase in volume, if Poisson's ratio is 0.25 and Young's modulus is 200 GPa.*

SOLUTION. Given : Length $(l) = 2 \text{ m} = 2 \times 10^3$ mm ; Width $(b) = 20$ mm ; Thickness $(t) = 15$ mm;

Tensile load $(P) = 30 \text{ kN} = 30 \times 10^3$ N ; Poisson's ratio $\left(\frac{1}{m}\right) = 0.25$ or $m = 4$ and Young's modulus of elasticity $(E) = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$.

Let δV = Increase in volume of the bar.

We know that original volume of the bar,

$$V = l.b.t = (2 \times 10^3) \times 20 \times 15 = 600 \times 10^3 \text{ mm}^3$$

and

$$\frac{\delta V}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right) = \frac{30 \times 10^3}{20 \times 15 \times (200 \times 10^3)}\left(1 - \frac{2}{4}\right) = 0.000\,25$$

$$\therefore \quad \delta V = 0.000\,25 \times V = 0.000\,25 \times (600 \times 10^3) = 150 \text{ mm}^3$$ **Ans.**

EXAMPLE 6.4. *A copper bar 250 mm long and 50 mm × 50 mm in cross-section is subjected to an axial pull in the direction of its length. If the increase in volume of the bar is 37.5 mm^3, find the magnitude of the pull. Take m = 4 and E = 100 GPa.*

SOLUTION. Given: Length (l) = 250 mm ; Width (b) = 50 mm ; Thickness (t) = 50 mm ; Increase in volume (δV) = 37.5 mm^3 ; (m) = 4 and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm^2.

Let P = Magnitude of the pull in kN.

We know that original volume of the copper bar,

$$V = l.b.t = (250 \times 50 \times 50) = 625 \times 10^3 \text{ mm}^3$$

and

$$\frac{\delta V}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right) = \frac{P}{50 \times 10 \times (100 \times 10^3)}\left(1 = \frac{2}{4}\right)$$

or

$$\frac{37.5}{625 \times 10^3} = \frac{P}{500 \times 10^6}$$

$$\therefore \quad P = \frac{37.5 \times (500 \times 10^6)}{625 \times 10^3} = 30 \times 10^3 \text{ N} = 30 \text{ kN} \qquad \textbf{Ans.}$$

EXAMPLE 2.1. *A steel rod 1 m long and 20 mm × 20 mm in cross-section is subjected to a tensile force of 40 kN. Determine the elongation of the rod, if modulus of elasticity for the rod material is 200 GPa.*

SOLUTION. Given : Length (l) = 1 m = 1×10^3 mm ; Cross-sectional area (A) = 20 × 20 = 400 mm^2 ; Tensile force (P) = 40 kN = 40×10^3 N and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

We know that elongation of the road,

$$\delta l = \frac{P.l}{A.E} = \frac{(40 \times 10^3) \times (1 \times 10^3)}{400 \times (20 \times 10^3)} = 0.5 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE 2.2. *A hollow cylinder 2 m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa.*

SOLUTION. Given : Length (l) = 2 m = 2×10^3 mm ; Outside diameter (D) = 50 mm ; Inside diameter (d) = 30 mm ; Load (P) = 25 kN = 25×10^3 N and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm^2.

Stress in the cylinder

We know that cross-sectional area of the hollow cylinder.

$$A = \frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4} \times [(50)^2 - (30)^2] = 1257 \text{ mm}^2$$

and stress in the cylinder,

$$\sigma = \frac{P}{A} = \frac{25 \times 10^3}{1257} = 19.9 \text{ N/mm}^2 = 19.9 \text{ MPa} \qquad \textbf{Ans.}$$

Deformation of the cylinder

We also know that deformation of the cylinder,

$$\delta l = \frac{P.l}{A.E} = \frac{(25 \times 10^3) \times (2 \times 10^3)}{1257 \times (100 \times 10^3)} = 0.4 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 2.3. *A load of 5 kN is to be raised with the help of a steel wire. Find the minimum diameter of the steel wire, if the stress is not to exceed 100 MPa.*

SOLUTION. Given : Load (P) = 5 kN = 5×10^3 N and stress (σ) = 100 MPa = 100 N/mm^2

Let d = Diameter of the wire in mm.

We know that stress in the steel wire (σ),

$$100 = \frac{P}{A} = \frac{5 \times 10^3}{\frac{\pi}{4} \times (d)^2} = \frac{6.366 \times 10^3}{d^2}$$

$$\therefore \quad d^2 = \frac{6.366 \times 10^3}{100} = 63.66 \quad \text{or} \quad d = 7.98 \text{ say } 8 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 2.4. *In an experiment, a steel specimen of 13 mm diameter was found to elongate 0.2 mm in a 200 mm gauge length when it was subjected to a tensile force of 26.8 kN. If the specimen was tested within the elastic range, what is the value of Young's modulus for the steel specimen ?*

SOLUTION. Given : Diameter $(d) = 13$ mm ; Elongation $(\delta l) = 0.2$ mm ; Length $(l) = 200$ mm and Force $(P) = 26.8$ kN.

Let E = Value of Young's modulus for the steel specimen.

We know that cross-sectional area of the specimen.

$$A = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (13)^2 = 132.73 \text{ mm}^2$$

and elongation of the specimen (δl)

$$0.2 = \frac{P.l}{A.E} = \frac{26.8 \times 20}{132.73\, E} = \frac{40.38}{E}$$

$$\therefore \quad E = \frac{40.38}{0.2} = 201.9 \text{ kN/mm}^2 = 201.9 \text{ GPa} \qquad \textbf{Ans.}$$

EXAMPLE 2.5. *A hollow steel tube 3.5 m long has external diameter of 120 mm. In order to determine the internal diameter, the tube was subjected to a tensile load of 400 kN and extension was measured to be 2 mm. If the modulus of elasticity for the tube material is 200 GPa, determine the internal diameter of the tube.*

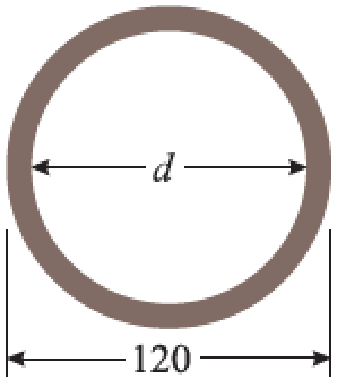


Fig. 2.3

SOLUTION. Given : Length (l) = 3.5 m = 3.5×10^3 mm ; External diameter (D) = 120 mm ; Load (P) = 400 kN = 400×10^3 N; Extension (δl) = 2 mm and modulus of elasticity E = 200 GPa = 200×10^3 N/mm^2.

Let d = Internal diameter of the tube in mm.

We know that area of the tube,

$$A = \frac{\pi}{4}[(120)^2 - d^2] = 0.7854\,[(120)^2 - d^2]$$

and extension of the tube (δl),

$$2 = \frac{P.l}{A.E} = \frac{(400\times 10^3)\times(3.5\times 10^3)}{0.7854\,[(120)^3 - d^2\,(200\times 10^3)} = \frac{8913}{14400 - d^2}$$

$$\therefore \quad 28800 - 2d^2 = 8913 \quad \text{or} \quad 2d^2 = 28800 - 8913 = 19887$$

$$\text{or} \quad d^2 = \frac{19887}{2} = 9943.5 \quad \text{or} \quad d = 99.71 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE 2.6. *Two wires, one of steel and the other of copper, are of the same length and are subjected to the same tension. If the diameter of the copper wire is 2 mm, find the diameter of the steel wire, if they are elongated by the same amount. Take E for steel as 200 GPa and that for copper as 100 GPa.*

SOLUTION. Given: Diameter of copper wire (d_C) = 2 mm ; Modulus of elasticity for steel (E_S) = 200 GPa = 200×10^3 N/mm^2 and modulus of elasticity for Copper (E_C) = 100 GPa = $100 \times 10_3$ N/mm^2.

Let d_S = Diameter of the steel wire,

l = Lengths of both the wires and

P = Tension applied on both the wires.

We know that area of the copper wire,

$$A_C = \frac{\pi}{4} \times (d_C)^2 = \frac{\pi}{4} \times (2)^2 = 3.142 \text{ mm}^2$$

and area of steel wire,

$$A_S = \frac{\pi}{4} \times (d_S)^2 = 0.7854\, d_S^2 \text{ mm}^2$$

We also know that increase in the length of the copper wire

$$\delta l_C = \frac{P\,l}{A_C\, E_C} = \frac{P\,l}{3.142 \times (100 \times 10^3)} = \frac{P\,l}{314.2 \times 10^3} \quad ...(i)$$

and increase in the length of the steel wire,

$$\delta l_S = \frac{P\,l}{A_S\, E_S} = \frac{P\,l}{0.7854\, d_S^2 \times (200 \times 10^3)} = \frac{P\,l}{157.1 \times 10^3 \times d_S^2} \quad ...(ii)$$

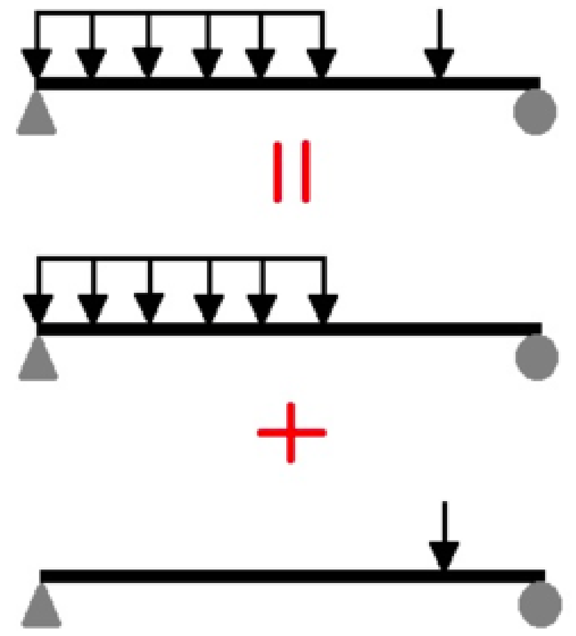
Since both the wires are elongated by the same amount, therefore equating equations (*i*) and (*ii*).

$$\frac{P\,l}{314.2 \times 10^3} = \frac{P\,l}{157.1 \times 10^3 \times d_S^2} \quad \text{or} \quad d_S^2 = \frac{314.2}{157.1} = 2$$

$$\therefore \quad d_S = \sqrt{2} = 1.41 \text{ mm} \quad \textbf{Ans.}$$

Principal of Superposition

- In physics and systems theory, the superposition principle, for all linear systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually. So that if input A produces response X and input B produces response Y then input (A + B) produces response (X + Y).



EXAMPLE 2.10. *A steel bar of cross-sectional area 200 mm² is loaded as shown in Fig. 2.6. Find the change in length of the bar. Take E as 200 GPa.*

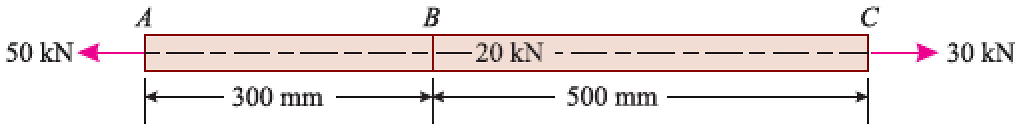


Fig. 2.6

Find the change in length of the bar. Take E as 200 GPa.

SOLUTION. Given: Cross-sectional area $(A) = 200\ \text{mm}^2$ and modulus of elasticity $(E) = 200$ GPa $= 200 \times 10^3\ \text{N/mm}^2$.

For the sake of simplification, the force of 50 kN acting at *A* may be split up into two forces of 20 kN and 30 kN respectively.

Now it will be seen that part *AB* of the bar is subjected to a tension of 20 kN and *AC* is subjected to a tension of 30 kN as shown in *Fig. 2.7.

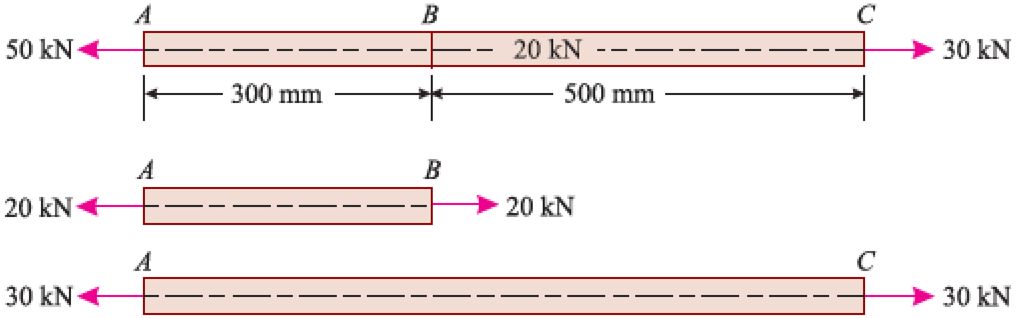


Fig. 2.7

We know that change in length of the bar.

$$\delta l = \frac{1}{AE}(P_1 l_1 + P_2 l_2)$$

$$= \frac{1}{200 \times 200 \times 10^3}\left[[(20 \times 10^3) \times (300)] + [(30 \times 10^3) \times (800)]\right] \text{mm}$$

$$= 0.75\ \text{mm} \qquad \textbf{Ans.}$$

EXAMPLE 2.11. *A brass bar, having cross-sectional area of 500 mm^2 is subjected to axial forces as shown in Fig. 2.8.*

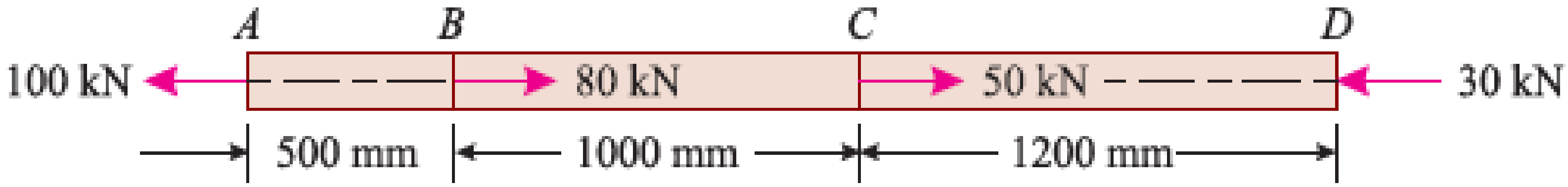


Fig. 2.8

Find the total elongation of the bar. Take E = 80 GPa.

SOLUTION. Given: Cross-sectional area (A) = 500 mm^2 and modulus of elasticity (E) = 80 GPa = 80 kN/mm^2.

For the sake of simplification, the force of 100 kN acting at A may be split up into two forces of 80 kN and 20 kN respectively. Similarly, the force of 50 kN acting at C may also be split up into two forces of 20 kN and 30 kN respectively.

Now it will be seen that the part AB of the bar is subjected to a tensile force of 80 kN, part AC is subjected to a tensile force of 20 kN and the part CD is subjected to a compression force of 30 kN as shown in Fig. 2.9.

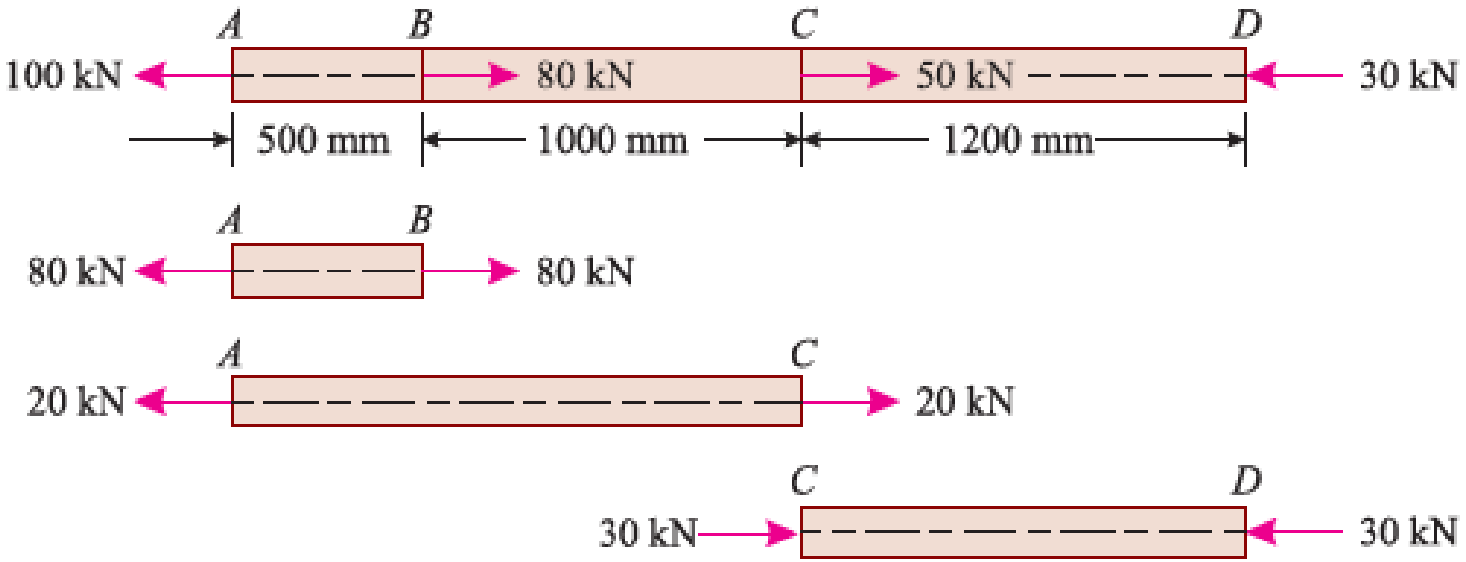


Fig. 2.9

We know that elongation of the bar,

$$\delta l = \frac{1}{AE}\left[P_1 l_1 + P_2 l_2 + P_3 l_3\right]$$

$$= \frac{1}{500 \times 80}\left[(80 \times 500) + (20 \times 1500) - (30 \times 1200)\right] \text{mm}$$

...(Taking plus sign for tension and minus for compression)

$$= 0.85 \text{ mm}$$ **Ans.**

EXAMPLE 2.12. *A steel rod ABCD 4.5 m long and 25 mm in diameter is subjected to the forces as shown in Fig. 2.10. If the value of Young's modulus for the steel is 200 GPa, determine its deformation.*

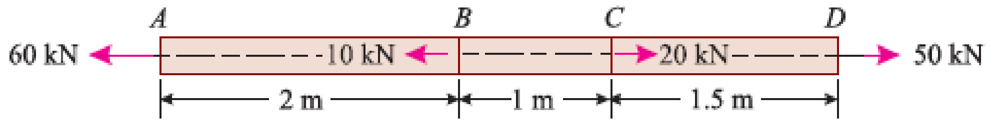


Fig. 2.10

SOLUTION. Given: Diameter (D) = 25 mm and Young's modulus (E) = 200 GPa = 200 kN/mm^2.

We know that cross-sectional area of the steel rod.

$$A = \frac{\pi}{4}(D)^2 = \frac{\pi}{4} \times (25)^2 = 491 \text{ mm}^2$$

For the sake of simplification, the force of 60 kN acting at A may be split up into two forces of 50 kN and 10 kN respectively. Similarly the force of 20 kN acting at C may also be split up into two forces of 10 kN and 10 kN respectively.

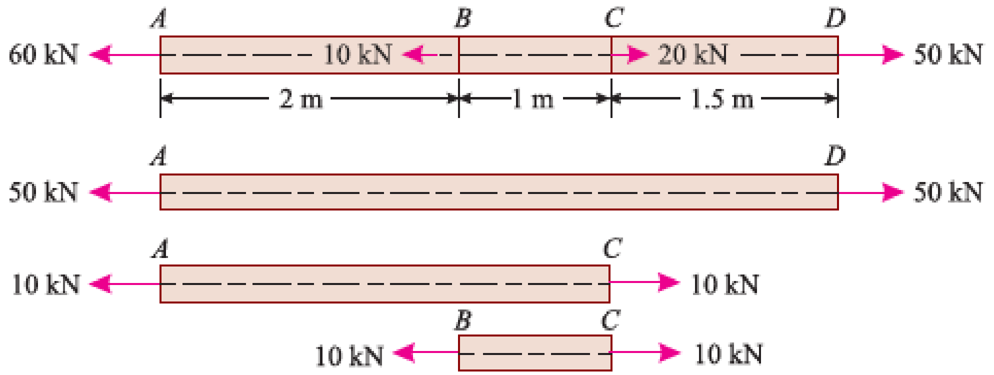


Fig. 2.11

Now it will be seen that the bar *AD* is subjected a tensile force of 50 kN, part *AC* is subjected to a tensile force of 10 kN and the part *BC* is subjected to a tensile force of 10 kN as shown in Fig. 2.11

We know that deformation of the bar,

$$\delta l = \frac{1}{AE}[P_1 l_1 + P_2 l_2 + P_3 l_3]$$

$$= \frac{1}{491 \times 200}\left[[50 \times (4.5 \times 10^3)] + [10 \times (3 \times 10^3)] + [10 \times (1 \times 10^3)\right] \text{mm}$$

$$= \frac{1}{491 \times 200} \times (265 \times 10^3) = 2.70 \text{ mm} \qquad \textbf{Ans.}$$

Stresses In the Bars of Different Sections

Sometimes a bar is made up of different lengths having different cross-sectional areas as shown in Fig. 3.1.

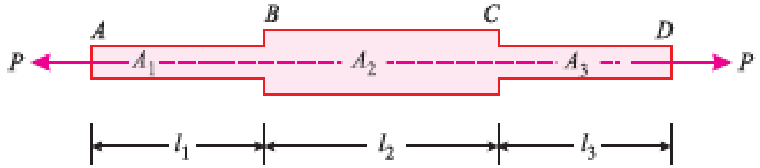


Fig. 3.1. Bars of different sections

In such cases, the stresses, strains and hence changes in lengths for each section is worked out separately as usual. The total changes in length is equal to the sum of the changes of all the individual lengths. It may be noted that each section is subjected to the same external axial pull or push.

Let P = Force acting on the body,

E = Modulus of elasticity for the body,

l_1 = Length of section 1,

A_1 = Cross-sectional area of section 1,

l_2, A_2 = Corresponding values for section 2 and so on.

We know that the change in length of section 1.

$$\delta l_1 = \frac{Pl_1}{A_1 E} \qquad \text{Similarly} \qquad \delta l_2 = \frac{Pl_2}{A_2 E} \qquad \text{and so on}$$

∴ Total deformation of the bar,

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 + \ldots\ldots\ldots\ldots$$

$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E} + \ldots\ldots\ldots\ldots$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} + \ldots\ldots\ldots\ldots\right)$$

Note. Sometimes, the modulus of elasticity is different for different sections. In such cases, the total deformation,

$$\delta l = P\left(\frac{l_1}{A_1 E_1} + \frac{l_2}{A_2 E_2} + \frac{l_3}{A_3 E_3} + \ldots\ldots\ldots\ldots\right)$$

EXAMPLE 3.2. *A member formed by connecting a steel bar to an aluminium bar is shown in Fig. 3.3.*

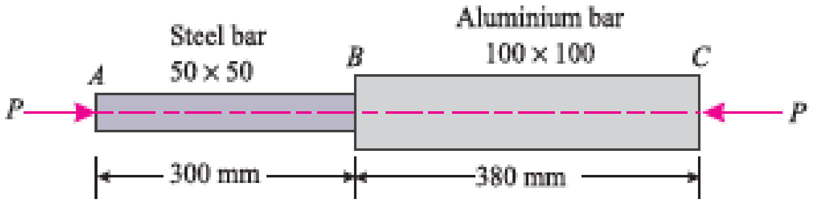


Fig. 3.3

Assuming that the bars are prevented from buckling sidewise, calculate the magnitude of force P, that will cause the total length of the member to decrease by 0.25 mm. The values of elastic modulus for steel and aluminium are 210 GPa and 70 GPa respectively.

SOLUTION. Given : Decrease in length (δl) = 0.25 mm ; Modulus of elasticity for steel (E_S) = 210 GPa = 210 × 10^3 N/mm^2 ; Modulus of elasticity for aluminium (E_A) = 70 GPa = 70 × 10^3 N/mm^2 ; Area of steel section (A_S) = 50 × 50 = 2 500 mm^2 ; Area of aluminium section (A_A) = 100 × 100 = 10000 mm^2 ; Length of steel section (l_S) = 300 mm and length of aluminium section (l_A) = 380 mm.

Let P = Magnitude of the force in kN.

We know that decrease in the length of the member (δl),

$$0.25 = P\left(\frac{l_S}{A_S\,E_S} + \frac{l_A}{A_A\,E_A}\right)$$

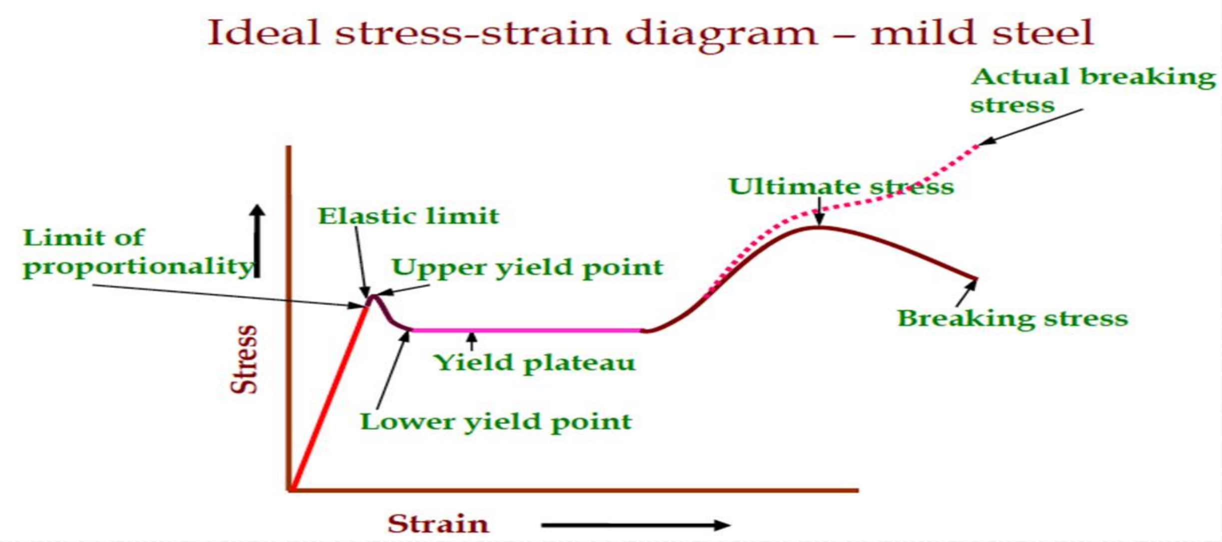
$$= P\left(\frac{300}{2500\times(210\times10^3)} + \frac{380}{10000\times(70\times10^3)}\right)$$

$$= \frac{780\,P}{700\times10^6}$$

$$\therefore \quad P = \frac{0.25\times(700\times10^6)}{780} = 224.4\times10^3\text{ N} = 224.4\text{ kN} \qquad \textbf{Ans.}$$

Factor of Safety

The ratio of elastic limit to the working stress (or ultimate stress to the working stress) is called the *factor of safety*. In the recent days, the general practice followed is, that for structural steel work (when subjected to gradually increasing loads) the factor of safety is taken as the ratio of elastic limit to the working stress; whose value is taken as 2 to 2.5. But in the case of cast iron, concrete, wood, etc. (or when structural steel work is subjected to sudden loads) the factor of safety is taken as the ratio of ultimate stress to the working stress, whose value is taken as 4 to 6.



EXAMPLE 36.1. *A mild steel rod of 12 mm diameter was tested for tensile strength, with the gauge length of 60 mm. Following were the observations :*

(*a*) *Final length = 78 mm*

(*b*) *Final diameter = 7 mm*

(*c*) *Yield load = 34 kN*

(*d*) *Ultimate load = 61 kN*

Calculate (a) yield stress, (b) ultimate tensile stress, (c) percentage reduction, and (d) percentage elongation.

SOLUTION.

Given. Original diameter of rod = 12 mm; Original length = 60 mm; Final length = 78 mm; Final diameter = 7 mm; Yield load = 34 kN = 3.4×10^4 N and ultimate load = 61 kN = 6.1×10^4 N.

$$\text{Original area} = \frac{\pi}{4} \times (12)^2 = 113 \text{ mm}^2$$

$$\text{Final area} = \frac{\pi}{4} \times (7)^2 = 38.5$$

Yield stress

We know that the yield stress

$$= \frac{\text{Yield load}}{\text{Area}} = \frac{3.4 \times 10^4}{113} \text{ N/mm}^2$$

$$= 300.8 \text{ N/mm}^2 \textbf{ Ans.}$$

Ultimate tensile stress

We know that the ultimate tensile stress

$$= \frac{\text{Ultimate load}}{\text{Area}} = \frac{6.1 \times 10^4}{113} \text{ N/mm}^2$$

$$= 539.8 \text{ N/mm}^2 \textbf{ Ans.}$$

Percentage reduction

We know that the percentage reduction

$$= \frac{\text{Original area} - \text{Final area}}{\text{Original area}} \times 100$$

$$= \frac{113 - 38.5}{113} \times 100 = 65.9\% \textbf{ Ans.}$$

Percentage elongation

We also know that the percentage elongation

$$= \frac{\text{Final length} - \text{Original length}}{\text{Final length}} \times 100$$

$$= \frac{78 - 60}{78} \times 100 = 23\% \textbf{ Ans.}$$

Determinate and Indeterminate beam

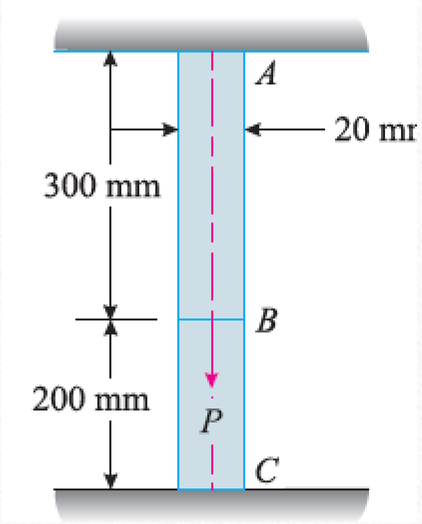
sometimes, the simple equations are not sufficient to solve such problems. Such problems are called statically indeterminate problems and the structures are called statically indeterminate structures.

Types of Statically Indeterminate Structures

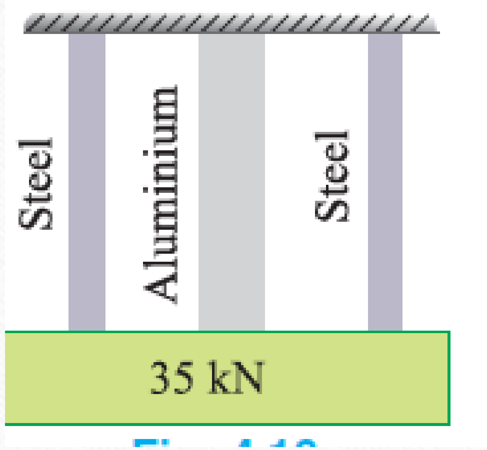
Though there are many types of statically indeterminate structures in the field of Strength of Materials yet the following are important from the subject point of view :

1. Simple statically indeterminate structures.
2. Indeterminate structures supporting a load.
3. Composite structures of equal lengths.
4. Composite structures of unequal lengths.

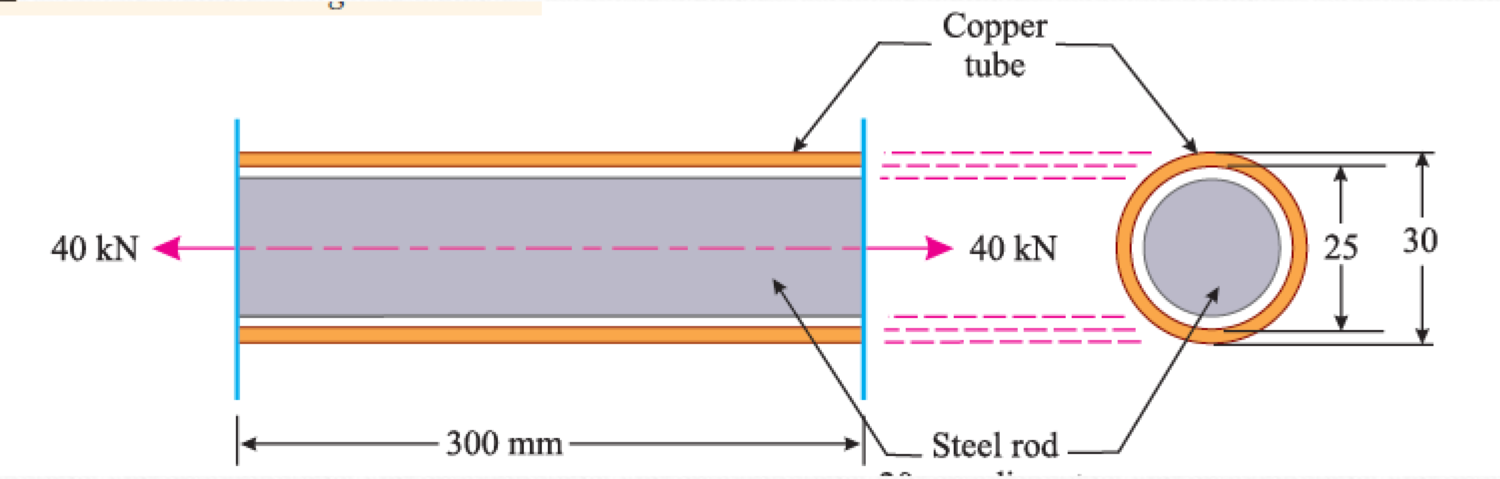
Now we shall study the procedures for the stresses and strains in the above mentioned indeterminate structures in the following pages. In order to solve the above mentioned types of statically indeterminate structures, we have to use different types of compatible equations.



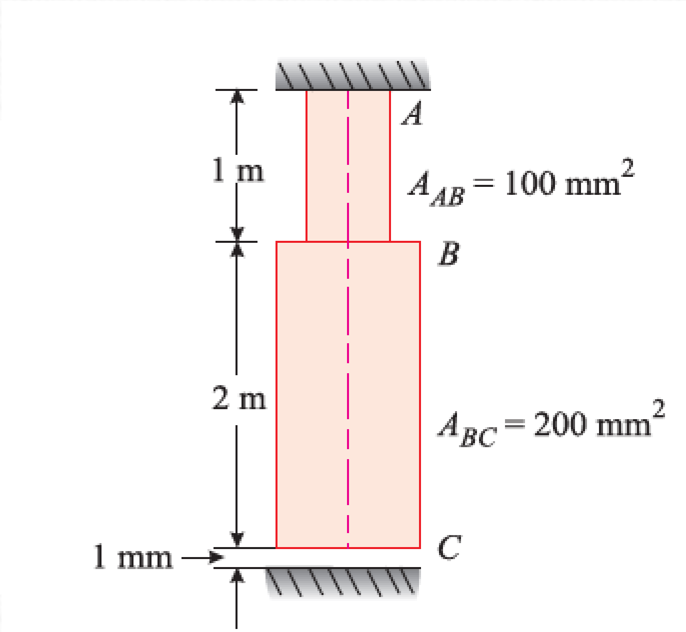
A
20 mr
300 mm
B
200 mm
P
C



Steel
Aluminium
Steel
35 kN



Copper tube
40 kN
40 kN
25
30
300 mm
Steel rod



A
1 m
$A_{AB} = 100\ mm^2$
B
2 m
$A_{BC} = 200\ mm^2$
C
1 mm

EXAMPLE 4.1. *A square bar of 20 mm side is held between two rigid plates and loaded by an axial force P equal to 450 kN as shown in Fig. 4.1.*

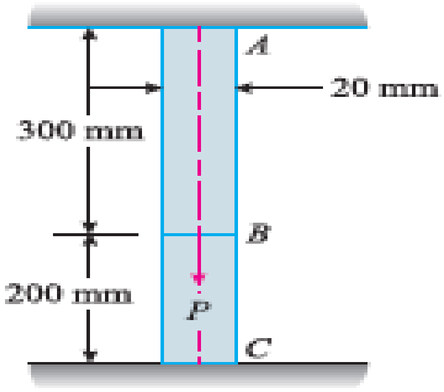


Fig. 4.1

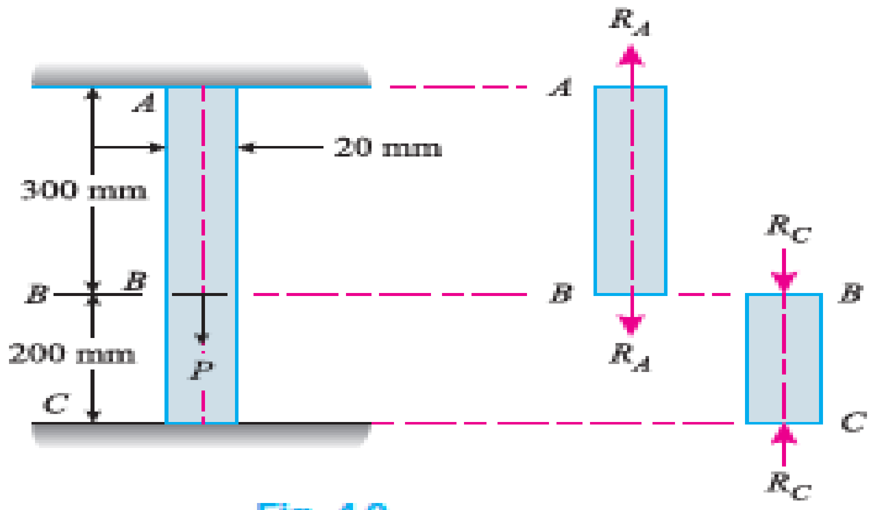


Fig. 4.2

Find the reactions at the ends A and C and the extension of the portion AB. Take E = 200 GPa.

SOLUTION. Given : Area of bar $(A) = 20 \times 20 = 400 \text{ mm}^2$; Axial force $(P) = 450 \text{ kN} = 450 \times 10^3$ N ; Modulus of elasticity $(E) = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$; Length of AB $(l_{AB}) = 300$ mm and length of BC $(l_{BC}) = 200$ mm.

Reaction at the ends

Let R_A = Reaction at A, and

R_C = Reaction at C.

Since the bar is held between the two rigid plates A and C, therefore, the upper portion will be subjected to tension, while the lower portion will be subjected to compression as shown in Fig. 4.2.

Moreover, the increase of portion AB will be equal to the decrease of the portion BC.

We know that sum of both the reaction is equal to the axial force, *i.e.*,

$$R_A + R_C = 450 \times 10^3 \quad \text{...(}i\text{)}$$

Increase in the portion AB,

$$\delta l_{AB} = \frac{R_A\, l_{AB}}{A\,E} = \frac{R_A \times 300}{A\,E}$$

and decrease in the portion BC,

$$\delta l_{BC} = \frac{R_C\, l_{BC}}{A\,E} = \frac{R_C \times 200}{A\,E} \quad \text{...(}ii\text{)}$$

Since the value δl_{AB} is equal to that of δl_{BC}, therefore equating the equations (*ii*) and (*iii*),

$$\frac{R_A \times 300}{A\,E} = \frac{R_C \times 200}{A\,E}$$

$$R_C = \frac{R_A \times 300}{200} = 1.5\,R_A$$

Now substituting the value of R_C in equation (*ii*),

$$R_A + 1.5\,R_A = 450 \quad \text{or} \quad 2.5\,R_A = 450$$

$$\therefore \quad R_A = \frac{450}{2.5} = 180 \text{ kN} \quad \textbf{Ans.}$$

and

$$R_C = 1.5\,R_A = 1.5 \times 180 = 270 \text{ kN} \quad \textbf{Ans.}$$

Extension of the portion AB

Substituting the value of R_A in equation (*ii*)

$$\delta_{AB} = \frac{R_A \times 300}{A\,E} = \frac{(180 \times 10^3) \times 300}{400 \times (200 \times 10^3)} = 0.675 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE 4.2. *An aluminium bar 3 m long and 2500 mm² in cross-section is rigidly fixed at A and D as shown in Fig. 4.3.*

Fig. 4.3

Determine the loads shared and stresses in each portion and the distances through which the points B and C will move. Take E for aluminium as 80 GPa.

SOLUTION. Given : Total length of bar (L) = 3 m ; Area of cross-section A = 2500 mm^2 ; Modulus of elasticity (E) = 80 GPa = 80×10^3 N/mm^2 and length of portion AB (l_{AB}) = l_{BC} = l_{CD} = 1 m = 1×10^3 mm.

Loads shared by each portion

Let P_{AB} = Load shared by the portion AB,

P_{BC} = Load shared by the portion BC and

P_{CD} = Load shared by the portion CD.

Since the bar is rigidly fixed at A and D, therefore the portion AB will be subjected to tension, while the portions BC and CD will be subjected to compression as shown in Fig. 4.4. Moreover, increase in the portion AB will be equal to the sum of the decreases in the portions BC and CD.

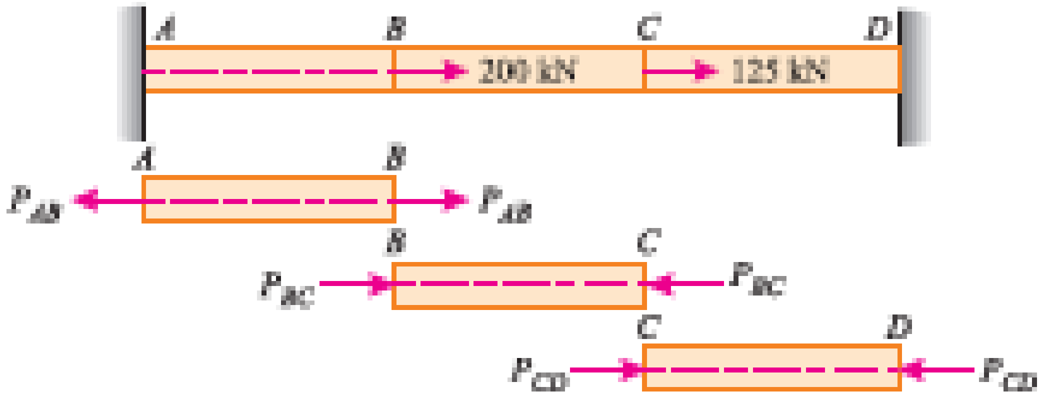


Fig. 4.4

From the geometry of the bar, we find that

$$P_{AB} + P_{BC} = 200 \quad \text{or} \quad P_{AB} = 200 - P_{BC} \qquad ...(i)$$

and

$$P_{CD} - P_{BC} = 125 \quad \text{or} \quad P_{CD} = 125 + P_{BC} \qquad ...(ii)$$

We know that increase in the length of portion AB,

$$\delta l_{AB} = \frac{P_{AB}\, l_{AB}}{A\,E} = \frac{P_{AB}\,(1 \times 10^3)}{A\,E} \qquad ...(iii)$$

Similarly, decrease in the length of portion BC,

$$\delta l_{BC} = \frac{P_{BC}\, l_{BC}}{A\,E} = \frac{P_{BC}\,(1 \times 10^3)}{A\,E} \qquad ...(iv)$$

and decrease in the length of portion CD,

$$\delta l_{CD} = \frac{P_{CD}\, l_{CD}}{AE} = \frac{P_{CD}(1\times 10^3)}{AE} \quad \ldots(v)$$

Since the value of δl_{AB} is equal to $\delta l_{BC} + \delta l_{CD}$, therefore

$$\frac{P_{AB}\times(1\times 10^3)}{AE} = \frac{P_{BC}\times(1\times 10^3)}{AE} + \frac{P_{CD}\times(1\times 10^3)}{AE}$$

$$\therefore \quad P_{AB} = P_{BC} + P_{CD}$$

Now substituting the values P_{AB} and P_{CD} from equations (*i*) and (*ii*) in the above equation,

$$(200 - P_{BC}) = P_{BC} + (125 + P_{BC})$$

$$\therefore \quad 3\,P_{BC} = 200 - 125 = 75 \text{ kN}$$

or $$P_{BC} = \frac{75}{3} = 25 \text{ kN}$$

$$\therefore \quad P_{AB} = 200 - P_{BC} = 200 - 25 = 175 \text{ kN} \quad \textbf{Ans.}$$

and $$P_{CD} = 125 + P_{BC} = 125 + 25 = 150 \text{ kN} \quad \textbf{Ans.}$$

Stresses in each portion

We know that stress in *AB*,

$$\sigma_{AB} = \frac{P_{AB}}{A} = \frac{175\times 10^3}{2500} = 70 \text{ N/mm}^2 = 70 \text{ MPa (tension)} \quad \textbf{Ans.}$$

Similarly, $$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{25\times 10^3}{2500} = 10 \text{ N/mm}^2 = 10 \text{ MPa (compression)} \quad \textbf{Ans.}$$

and $$\sigma_{CD} = \frac{P_{CD}}{A} = \frac{150\times10^3}{2500} = 60\ \text{N/mm}^2 = 60\ \text{MPa (compression)}\quad \textbf{Ans.}$$

Distance through which the points B and C will move

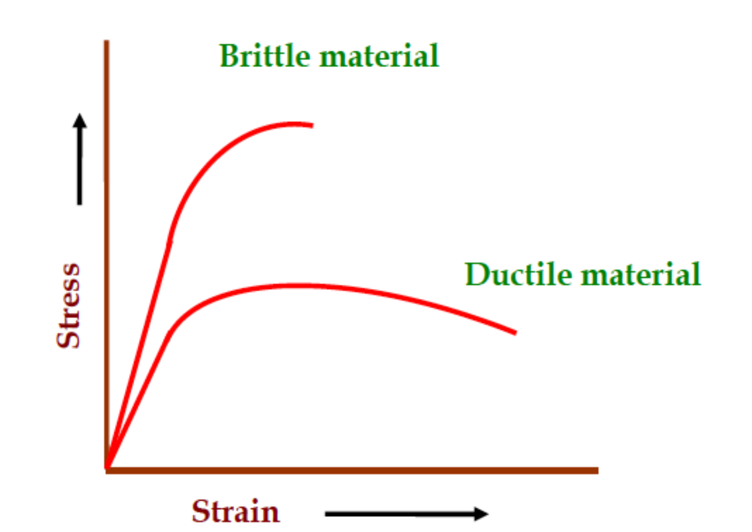
Substituting the value of P_{AB} in equation (*iii*), we get

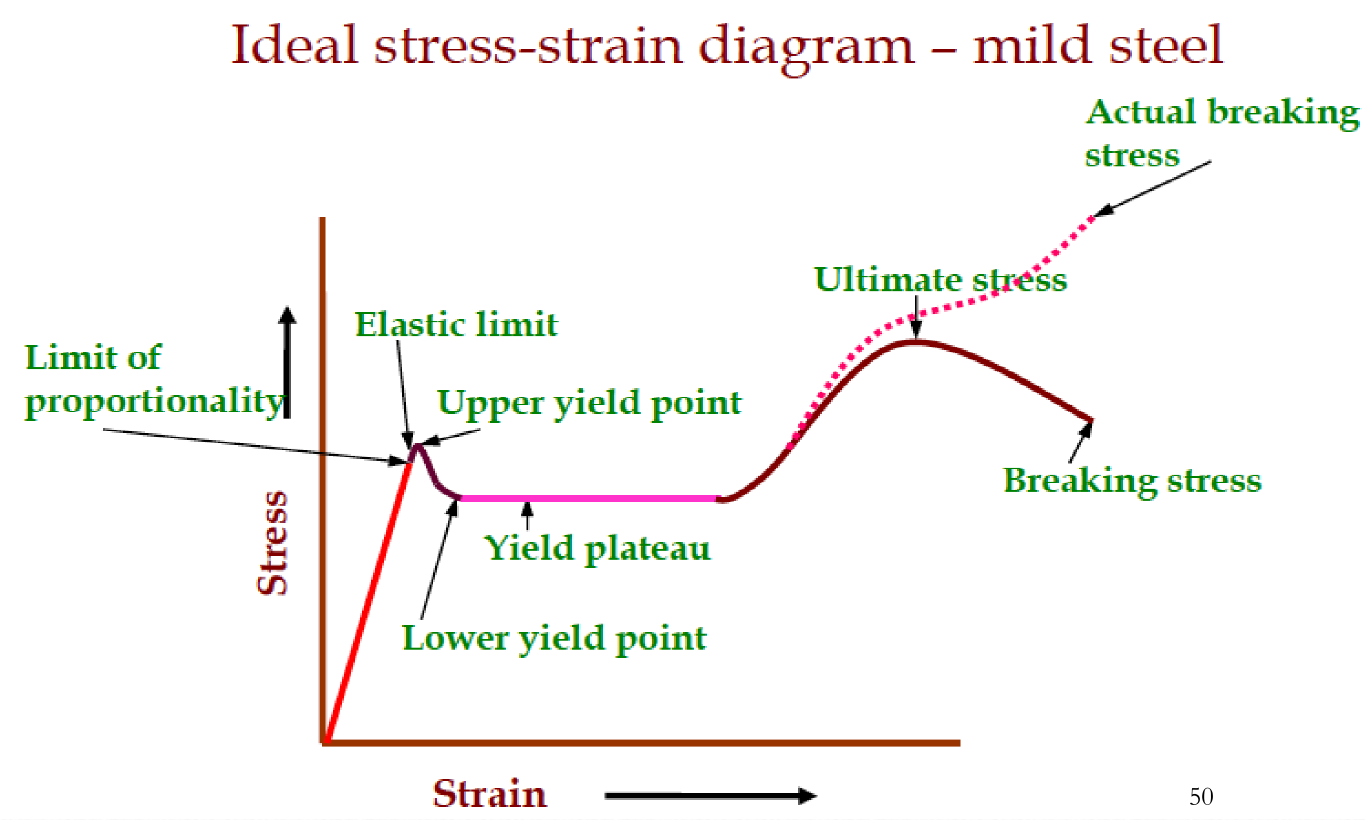
$$\delta l_{AB} = \frac{P_{AB}\times l_{AB}}{A\,E} = \frac{175\times10^3\times(1\times10^3)}{2500\times(80\times10^3)} = 0.875\ \text{mm}\qquad \textbf{Ans.}$$

and now substituting the value of P_{CD} in equation (*iv*), we get

$$\delta l_{CD} = \frac{P_{CD}\times l_{CD}}{A\,E} = \frac{(150\times10^3)\times(1\times10^3)}{2500\times(80\times10^3)} = 0.75\ \text{mm}\qquad \textbf{Ans.}$$

Stress-Strain Diagram and Modulus of Elasticity and Modulus of Rigidity, Bulk Modulus. Interrelation between elastic constants,





Modulus of Elasticity or Young's Modulus (E)

We have already discussed that whenever a material is loaded, within its elastic limit, the stress is proportional to strain. Mathematically stress,

$$\sigma \propto \varepsilon$$

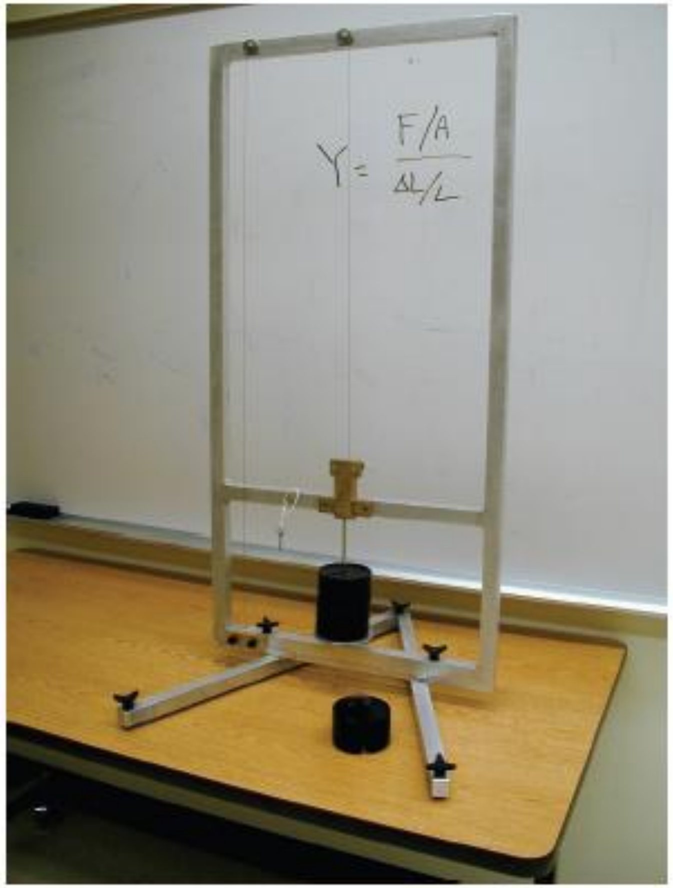
$$= E \times \varepsilon$$

or $$E = \frac{\sigma}{\varepsilon}$$

σ = Stress,

ε = Strain, and

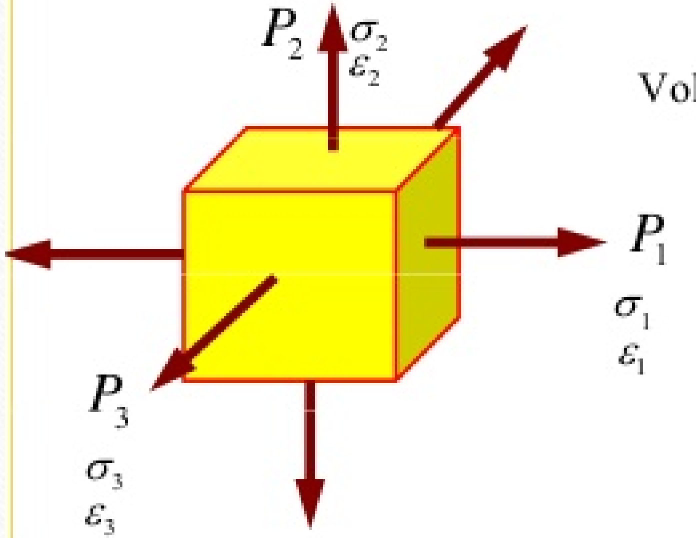
E = A constant of proportionality known as modulus of elasticity or Young's modulus. Numerically, it is that value of tensile stress, which when applied to a uniform bar will increase its length to double the original length if the material of the bar could remain perfectly elastic throughout such an excessive strain.



Young's Modulus Appratus

Volumetric Strain

- Three mutually perpendicular normal stresses



Volumetric strain $\varepsilon_v = \frac{\delta V}{V} \cong \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

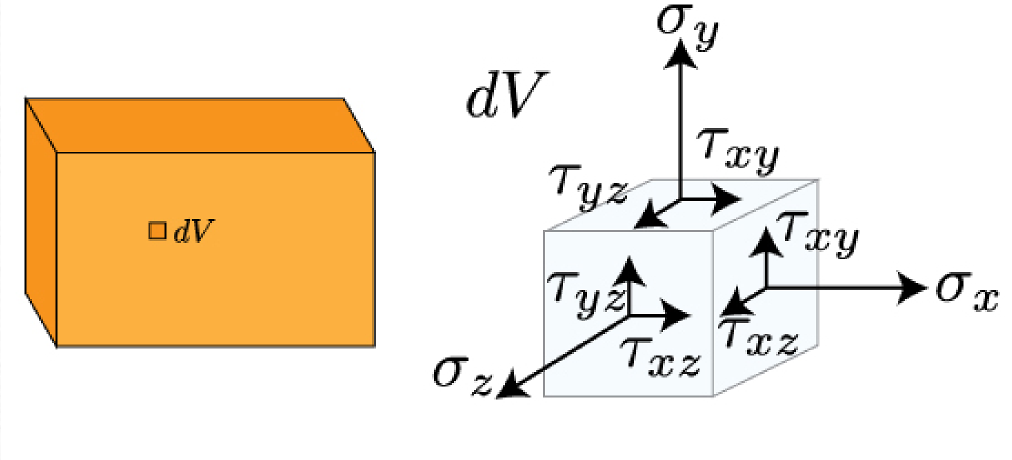
$$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E}$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\therefore \varepsilon_v = \frac{\delta V}{V} \cong \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (1 - 2\mu)\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{E}\right)$$

Stress-Strain Relation

Generalized Hooke's Law



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

Bulk Modulus

A very common type of stress that causes dilation is known as hydrostatic stress. This is just simply a pressure that acts equally on the entire material. Since it is acting equally, that means:

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$p = \sigma_x = \sigma_y = \sigma_z$$

$$e = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

$$e = \frac{3(1-2\nu)}{E} p$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$K = \frac{E}{3(1-2\nu)}$$

Shear Modulus or Modulus of Rigidity

It has been experimentally found that within the elastic limit, the shear stress is proportional to the shear strain. Mathematically

$$\tau \propto \phi$$

or $$\tau = C \times \phi$$

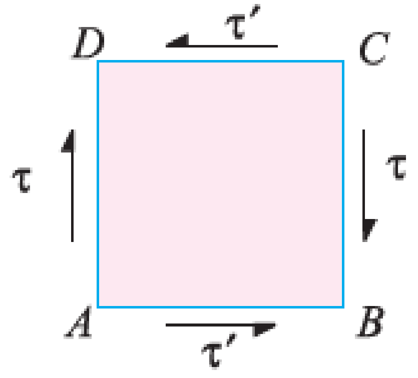
or $$\frac{\tau}{\phi} = C \text{ (or } G \text{ or } N\text{)}$$

where τ = Shear stress,

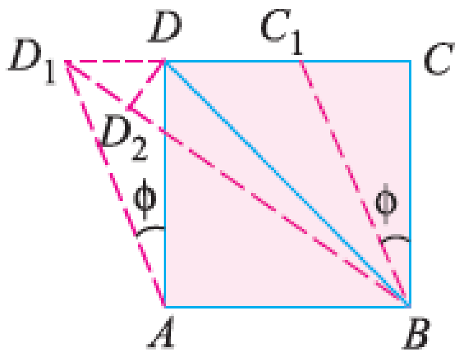
ϕ = Shear strain, and

C = A constant, known as shear modulus or modulus of rigidity. It is also denoted by G or N.

Relation Between Modulus of Elasticity and Modulus of Rigidity



(*a*) Before distortion



(*b*) After distortion

Fig. 6.11

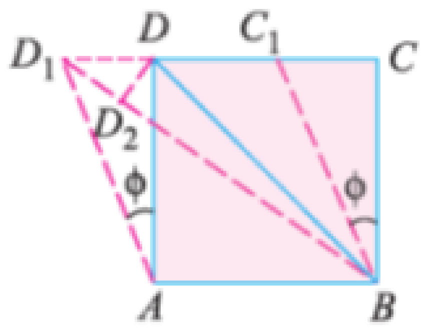
Consider a cube of length l subjected to a shear stress of τ as shown in Fig. 6.11 (*a*). A little consideration will show that due to these stresses the cube is subjected to some distortion, such that the diagonal *BD* will be elongated and the diagonal *AC* will be shortened. Let this shear stress t cause shear strain ϕ as shown in Fig. 6.11 (*b*). We see that the diagonal *BD* is now distorted to BD_1.

$$\therefore \qquad \text{Strain of } BD = \frac{BD_1 - BD}{BD} \qquad \ldots\left(\because \text{ Strain} = \frac{\delta l}{l}\right)$$

$$= \frac{D_1 D_2}{BD} = \frac{DD_1 \cos 45^\circ}{AD\sqrt{2}} = \frac{DD_1}{2\,AD} = \frac{\phi}{2}$$

Thus we see that the linear strain of the diagonal *BD* is half of the shear strain and is tensile in nature. Similarly it can be proved that the linear strain of the diagonal *AC* is also equal to half of the shear strain, but is compressive in nature. Now this linear strain of the diagonal *BD*.

$$= \frac{\phi}{2} = \frac{\tau}{2C} \qquad \ldots(i)$$



where τ = Shear stress and

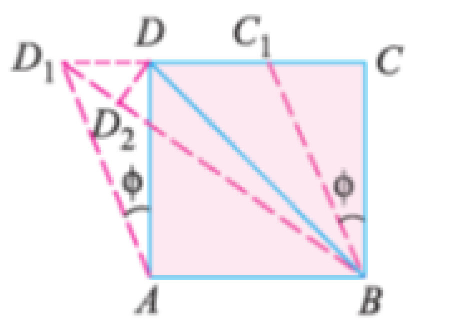
C = Modulus of rigidity.

Let us now consider this shear stress t acting on the sides *AB*, *CD*, *CB* and *AD*. We know that the effect of this stress is to cause tensile stress on the diagonal *BD* and compressive stress on the diagonal *AC*. Therefore tensile strain on the diagonal *BD* due to tensile stress on the diagonal *BD*

$$= \frac{\tau}{E} \qquad \ldots(ii)$$

and the tensile strain on the diagonal *BD* due to compressive stress on the diagonal *AC*

$$= \frac{1}{m} \times \frac{\tau}{E}$$



The combined effect of the above two stresses on the diagonal *BD*

$$= \frac{\tau}{E} + \frac{1}{m} \times \frac{\tau}{E} = \frac{\tau}{E}\left(1 + \frac{1}{m}\right) = \frac{\tau}{E}\left(\frac{m+1}{m}\right)$$

Equating equations (*i*) and (*iv*),

$$\frac{\tau}{2C} = \frac{\tau}{E}\left(\frac{m+1}{m}\right) \quad \text{or} \quad C = \frac{mE}{2(m+1)}$$

$$G = \frac{E}{2(1+\nu)}$$

Interrelation between elastic constants

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$\frac{E-2G}{2G} = \frac{3K-E}{6K}$$

EXAMPLE 6.9. *If the values of modulus of elasticity and Poisson's ratio for an alloy body is 150 GPa and 0.25 respectively, determine the value of bulk modulus for the alloy.*

SOLUTION. Given: Modulus of elasticity (E) = 150 GPa = 150×10^3 N/mm^2 and Poisson's ratio

$$\left(\frac{1}{m}\right) = 0.25 \qquad \text{or} \qquad m = 4.$$

We know that value of the bulk modulus for the alloy,

$$K = \frac{mE}{3(m-2)} = \frac{4 \times (150 \times 10^3)}{3(4-2)} = 100 \times 10^3 \text{ N/mm}^2$$

$$= 100 \text{ GPa} \qquad \textbf{Ans.}$$

EXAMPLE 6.10. *For a given material, Young's modulus is 120 GPa and modulus of rigidity is 40 GPa. Find the bulk modulus and lateral contraction of a round bar of 50 mm diameter and 2.5 m long, when stretched 2.5 mm. Take poisson's ratio as 0.25.*

SOLUTION. Given : Young's modulus (E) = 120 GPa = 120×10^3 N/mm^2 ; Modulus of rigidity (C) = 40 GPa = 40×10^3 N/mm^2 ; Diameter (d) = 50 mm ; Length (l) = 2.5 m = 2.5×10^3 mm ; Linear stretching or change in length (δl) = 2.5 mm and poisson's ratio = 0.25 or m = 4.

Bulk modulus of the bar

We know that bulk modulus of the bar,

$$K = \frac{mE}{3(m-2)} = \frac{4 \times (120 \times 10^3)}{3(4-2)} = 80 \times 10^3 \text{ N/mm}^2$$

$$= 80 \text{ GPa} \qquad \textbf{Ans.}$$

Lateral contraction of the bar

Let δd = Lateral contraction of the bar (or change in diameter)

We know that linear strain,

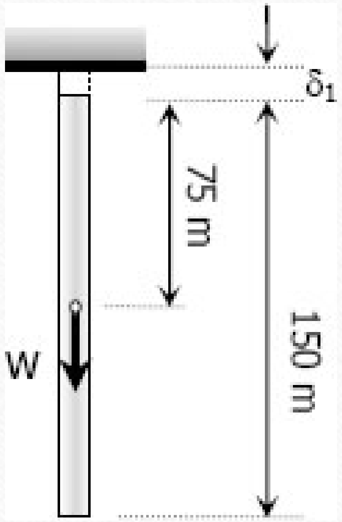
$$\varepsilon = \frac{\delta l}{l} = \frac{2.5}{2.5 \times 10^3} = \frac{1}{1000} = 0.001$$

and lateral strain, $\frac{\delta d}{d} = \frac{1}{m} \times \varepsilon = 0.25 \times 0.001 = 0.25 \times 10^{-3}$

$\therefore \quad \delta d = d \times (0.25 \times 10^{-3}) = 50 \times (0.25 \times 10^{-3}) = 0.0125 \text{ mm}$ **Ans.**

Axial Deformation along with Self weight

- A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and $E = 200 \times 10^3\ MN/m^2$, find the total elongation of the rod.

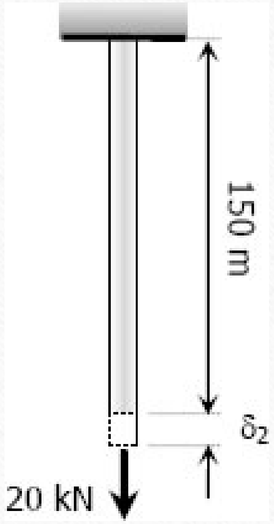


Elongation due to its own weight:

δ1=PL/ AE

δ1=3465.3825(75000)300(200000)

δ1=4.33 mm



Elongation due to applied load:

δ2=PL/AE

Thus,

δ2=20000(150000)/300(200000)

δ2=50 mm

Total elongation:

δ=δ1+δ2

δ=4.33+50=54.33 mm

Composite Sections

Bars in Series

- A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. p-211. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.

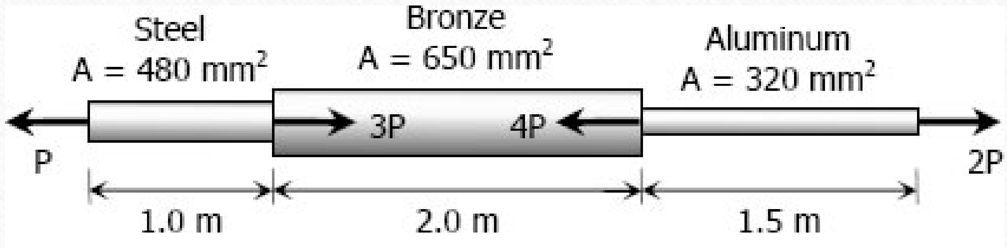
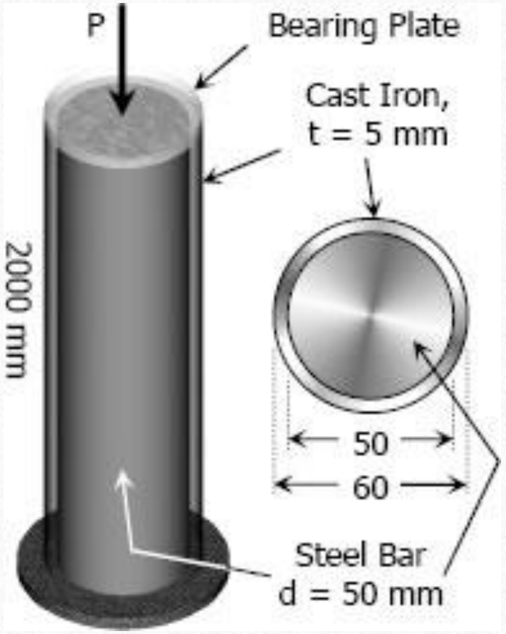


Figure P -211

Use the smallest value of P, **P = 12.8 kN**

Composite Bar (Bars in Parallel)

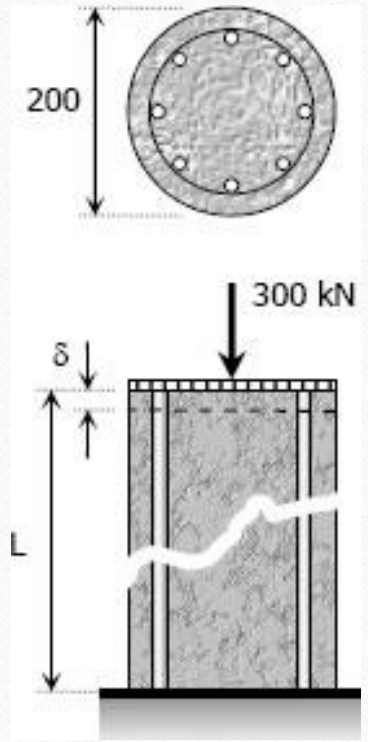
- A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.



P=191.64kNP=191.64kN

Composite Bar (Bars in Parallel)

A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.



A reinforced concrete circular section of 50,000 mm² cross-sectional area carries 6 reinforcing bars whose total area is 500 mm². Find the safe load, the column can carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.

SOLUTION. Given : Area of column = 50,000 mm² ; No. of reinforcing bars = 6 ; Total area of steel bars (A_S) = 500 mm² ; Max stress in concrete (σ_C) = 3.5 MPa = 3.5 N/mm² and modular ratio $\left(\frac{E_S}{E_C}\right)$ = 18.

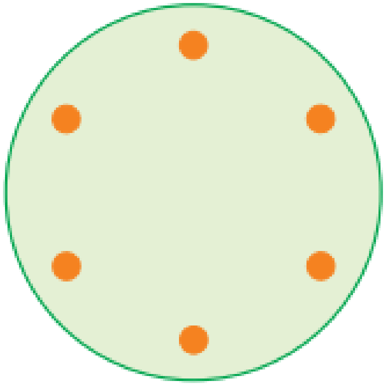


Fig. 3.22

We know that area of concrete,

$$A_C = 50,000 - 500 = 49,500 \text{ mm}^2$$

and stress in steel,

$$\sigma_S = \frac{E_S}{E_C} \times \sigma_C = 18 \times 3.5 = 63 \text{ N/mm}^2$$

∴ Safe load, the column can carry,

$$P = (\sigma_S \cdot A_S) + (\sigma_C \cdot A_C) = (63 \times 500) + (3.5 \times 49,500) \text{ N}$$

$$= 204\ 750 \text{ N} = 204.75 \text{ kN} \qquad \textbf{Ans.}$$

A reinforced concrete column 500 mm × 500 mm in section is reinforced with 4 steel bars of 25 mm diameter, one in each corner. The column is carrying a load of 1000 kN. Find the stresses in the concrete and steel bars. Take E for steel = 210 GPa and E for concrete = 14 GPa.

SOLUTION. Given : Area of column = 500 × 500 = 2,50,000 mm^2; No. of steel bars (n) = 4 ; Diameter of steel bars (d) = 25 mm ; Load on column (P) = 1,000 kN = $1{,}000 \times 10^3$ N ; Modulus of elasticity of steel (E_S) = 210 GPa and modulus of elasticity of concrete (E_C) = 14 GPa.

Let σ_S = Stress in steel, and

σ_C = Stress in concrete.

We know that area of steel bars,

$$A_S = 4 \times \frac{\pi}{4} \times (d)^2 \text{ mm}^2 \quad \text{...(i)}$$

$$= 4 \times \frac{\pi}{4} \times (25)^2 = 1963 \text{ mm}^2$$

$\therefore$ Area of concrete, $A_C = 250,000 - 1963 \text{ mm}^2$

$$= 248\,037 \text{ mm}^2$$

We also know that stress in steel,

$$\sigma_S = \frac{E_S}{E_C} \times \sigma_C = \frac{210}{14} \times \sigma_C = 15\,\sigma_C \quad \ldots(ii)$$

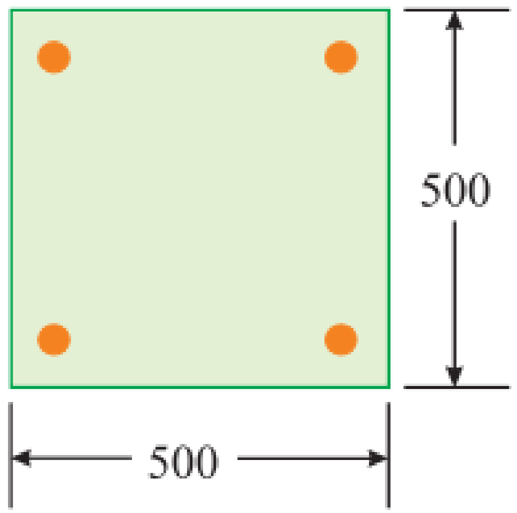


Fig. 3.23

and total load (P), $1{,}000 \times 10^3 = (\sigma_S \,.\, A_S) + (\sigma_C \,.\, A_C)$

$$= (15\,\sigma_C \times 1963) + (\sigma_C \times 248\,037) = 277\,482\,\sigma_C$$

$$\sigma_C = \frac{1{,}000 \times 10^3}{277\,482} = 3.6 \text{ N/mm}^2 = 3.6 \text{ MPa} \qquad \textbf{Ans.}$$

and $\sigma_S = 15\,\sigma_C = 15 \times 3.6 = 54 \text{ MPa}$ **Ans.**

A reinforced concrete circular column of 400 mm diameter has 4 steel bars of 20 mm diameter embeded in it. Find the maximum load which the column can carry, if the stresses in steel and concrete are not to exceed 120 MPa and 5 MPa respectively. Take modulus of elasticity of steel as 18 times that of concrete.

SOLUTION. Given : Diameter of column (D) = 400 mm ; No. of reinforcing bars = 4 ; Diameter of bars (d) = 20 mm ; Maximum stress in steel($\sigma_{S(max)}$) = 120 MPa = 120 N/mm^2 ; Maximum stress in concrete ($\sigma_{C(max)}$) = 5 MPa = 5 N/mm^2 and modulus of elasticity of steel (E_S) = 18 E_C.

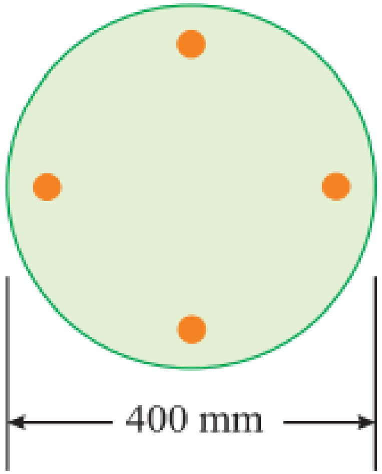


Fig. 3.24

We know that total area of the circular column.

$$= \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (400)^2 = 125\,660 \text{ mm}^2$$

and area of reinforcement (*i.e.*, steel),

$$A_S = 4 \times \frac{\pi}{4} \times (d)^2 = 4 \times \frac{\pi}{4} \times (20)^2 \text{ mm}^2$$

$$= 1257 \text{ mm}^2$$

$\therefore$ Area of concrete,

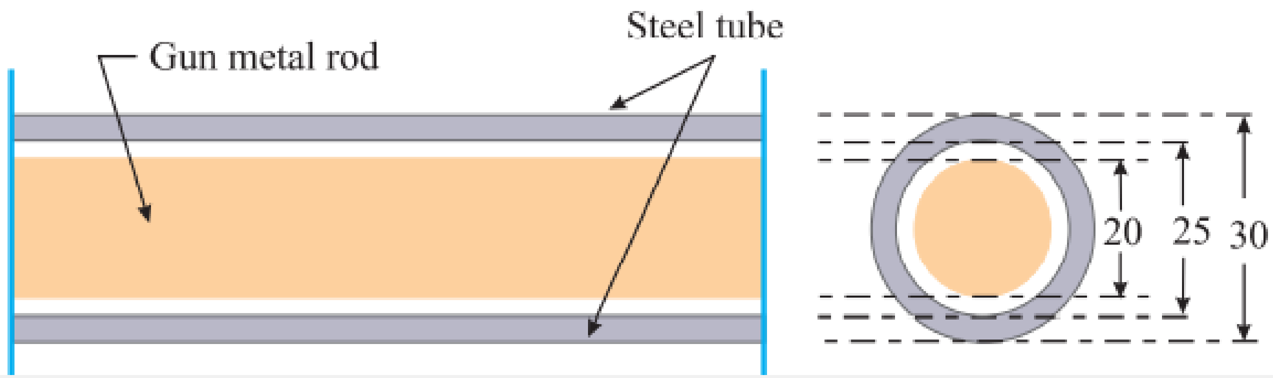
$$A_C = 125\,660 - 1257 = 124\,403 \text{ mm}^2$$

First of all let us find out the maximum stresses developed in the steel and concrete. We know that if the stress in steel is 120 N/mm^2, then stress in the concrete.

A gun metal rod 20 mm diameter, screwed at the ends, passes through a steel tube 25 mm and 30 mm internal and external diameters respectively. The nuts on the rod are screwed tightly home on the ends of the tube. Find the intensity of stress in each metal, when the common temperature rises by 200°F. Take.

Coefficient of expansion for steel	=	*6 × 10–6/°F*
Coefficient of expansion for gun metal	=	*10×10^{-6}/°F*
Modulus of elasticity for steel	=	*200 GPa*
Modulus of elasticity for gun metal	=	*100 GPa.*

SOLUTION. Given : Diameter of gun metal rod = 20 mm ; Internal diameter of steel tube = 25 mm; External diameter of steel tube = 30 mm ; Rise in temperature (t) = 200°F ; Coefficient of expansion for steel (α_S) = 6×10^{-6}/°F ; Coefficient of expansion for gun metals (α_G) = 10×10^{-6}/°F; Modulus of elasticity for steel (E_S) = 200 GPa = 200×10^3 N/mm^2 and modulus of elasticity for gun metal (E_G) = 100 GPa = 100×10^3 N/mm^2.



We know that area of gun metal rod,

$$A_G = \frac{\pi}{4} \times (20)^2 = 100\,\pi \text{ mm}^2$$

and area of steel tube $$A_S = \frac{\pi}{4} [(30)^2 - (25)^2] = 68.75\,\pi \text{ mm}^2$$

We also know that when the common temperature of the gun metal rod and steel tube will increase, the free expansion of gun metal rod will be more than that of steel tube (because α_G is greater than α_S). Thus the gun metal rod will be subjected to compressive stress and the steel tube will be subjected to tensile stress. Since the tensile load on the steel tube is equal to the compressive load on the gun metal rod, therefore stress in steel,

$$\sigma_S = \frac{A_G}{A_S} \times \sigma_S = \frac{100\,\pi}{68.75\,\pi} \times \sigma_G = 1.45\,\sigma_G$$

We know that strain in steel tube,

$$\varepsilon_S = \frac{\sigma_S}{E_S} = \frac{\sigma_S}{200 \times 10^3}$$

and $$\varepsilon_G = \frac{\sigma_G}{E_G} = \frac{\sigma_G}{100 \times 10^3}$$

We also know that total strain,

$$\varepsilon_S + \varepsilon_G = t\,(\alpha_G - \alpha_S)$$

We also know that total strain,

$$\varepsilon_S + \varepsilon_G = t\,(\alpha_G - \alpha_S)$$

$$\frac{\sigma_S}{200 \times 10^3} + \frac{\sigma_G}{100 \times 10^3} = 200\,[(10 \times 10^{-6}) - (6 \times 10^{-6})]$$

$$\frac{1.45\sigma_G}{200 \times 10^3} + \frac{\sigma_G}{100 \times 10^3} = 200 \times (4 \times 10^{-6})$$

$$\frac{3.45\sigma_G}{200 \times 10^3} = 800 \times 10^{-6}$$

$$3.45\ \sigma G = (800 \times 10^{-6}) \times (200 \times 10^3) = 160$$

$$\therefore \quad \sigma_G = \frac{160}{3.45} = 46.4\ \text{N/mm}^2 = 46.4\ \text{MPa} \qquad \textbf{Ans.}$$

and $$\sigma_S = 1.45\ \sigma_G = 1.45 \times 46.4 = 67.3\ \text{MPa} \qquad \textbf{Ans.}$$

Bars in Parallel

- A rigid block of mass M is supported by three symmetrically spaced rods as shown in Fig. P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

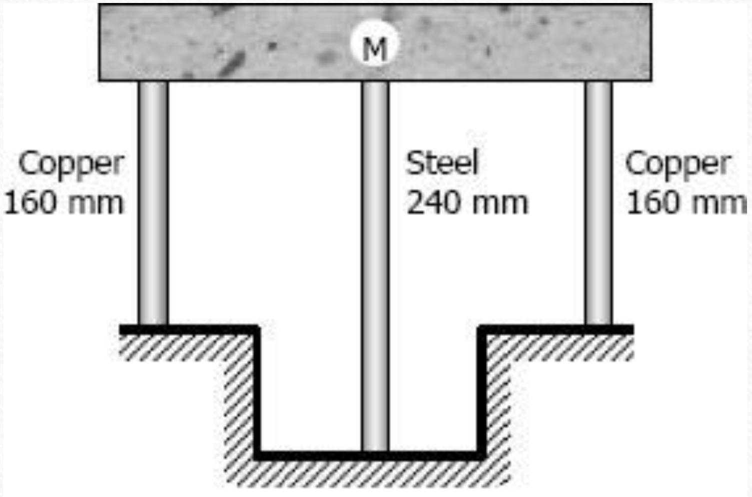


Figure P-236 and P-237

M=22358.4 kg

Bars in parallel inclined

- As shown in Fig. P-254, a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially stress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.

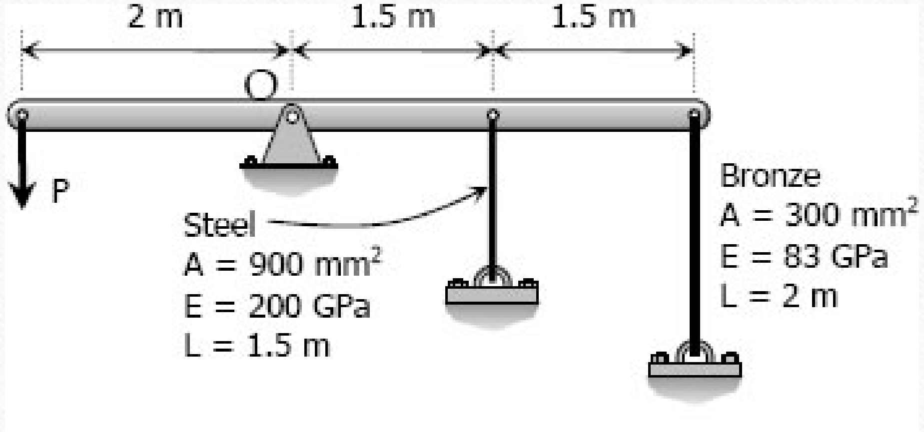
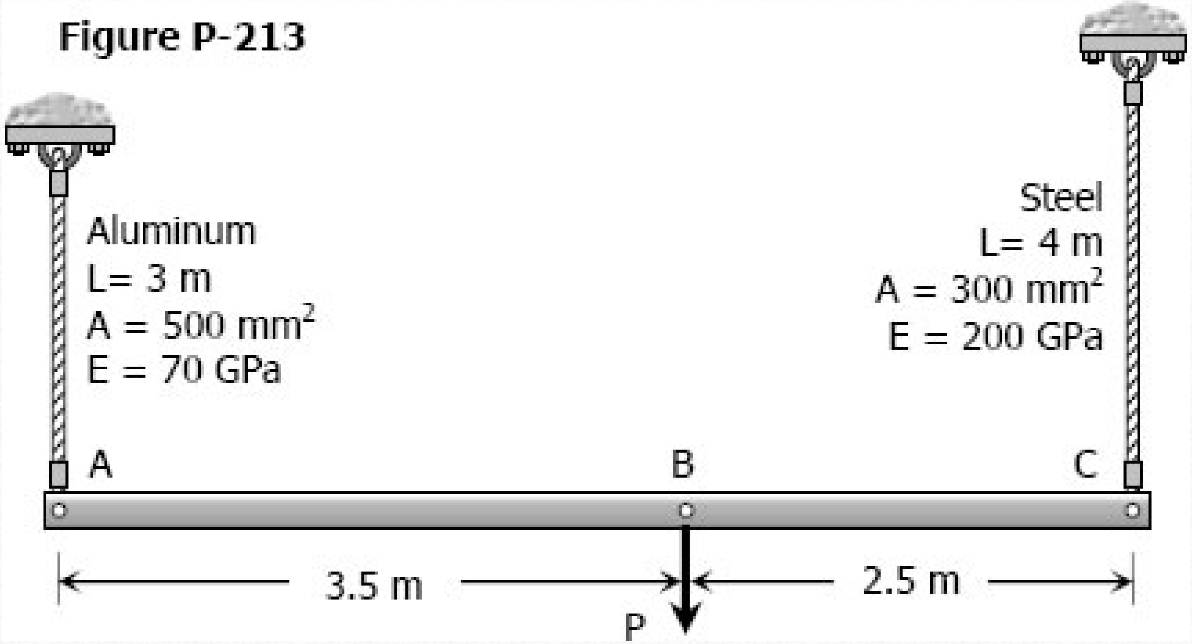


Figure P-254

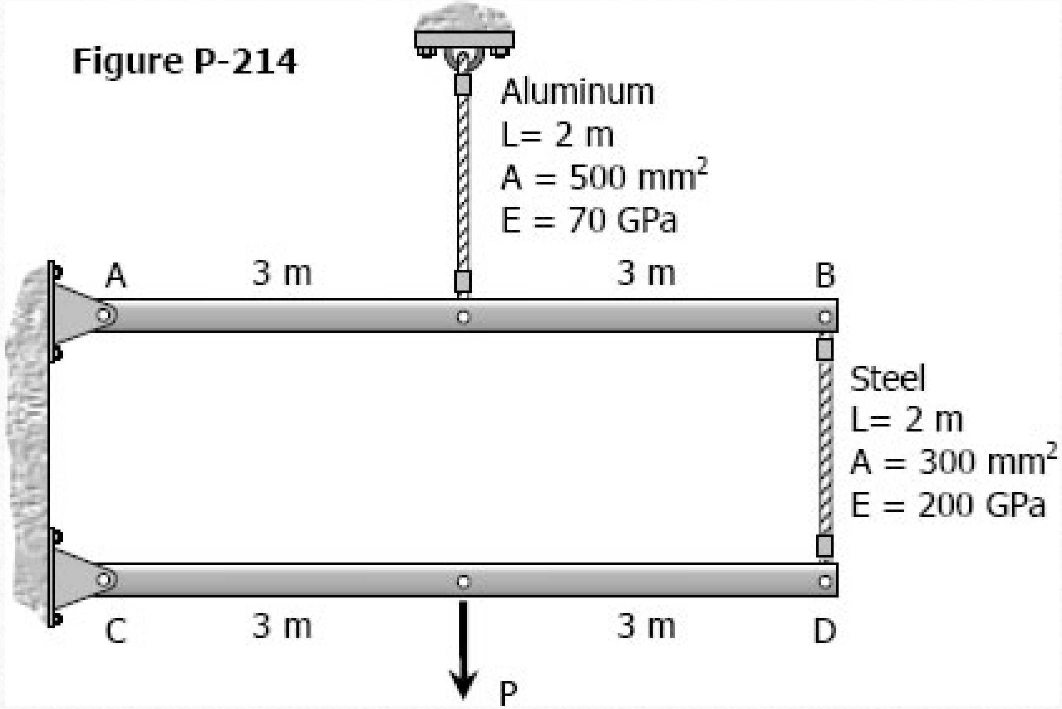
Bars in Parallel

The rigid bar AB, attached to two vertical rods as shown in Fig. P-213, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.



$\delta B = 1.87mm$

Bars in parallel



- The rigid bars AB and CD shown in Fig. P-214 are supported by pins at A and C and the two rods. Determine the maximum force P that can be applied as shown if its vertical movement is limited to 5 mm. Neglect the weights of all members.

Thermal Stress

Temperature changes cause the body to expand or contract

$$\delta_T = \alpha L (T_f - T_i) = \alpha L \Delta T$$

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure.

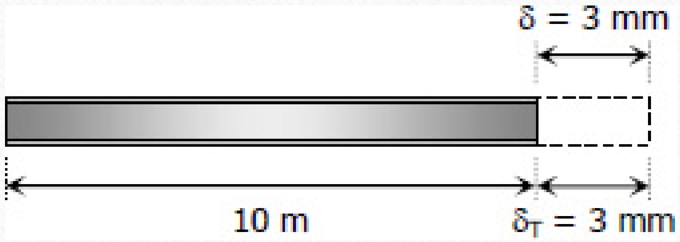
In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

$$\delta_T = \alpha L \Delta T$$

$$\sigma = E \alpha \Delta T$$

Thermal Stresses in Single Material

- Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume $\alpha = 11.7\ \mu m/(m\cdot°C)$ and $E = 200$ GPa.



EXAMPLE 5.1. *A aluminium alloy bar, fixed at its both ends is heated through 20 K. Find the stress developed in the bar. Take modulus of elasticity, and coefficient of linear expansion for the bar material as 80 GPa and 24 × 10^{-6}/K respectively.*

SOLUTION. Given : Increase in temperature (t) = 20 K ; Modulus of elasticity (E) = 80 GPa = 80 × 10^3 N/mm^2 and Coefficient of linear expansion (α) = 24 × 10^{-6}/K

We know that thermal stress developed in the bar,

$$\sigma = \alpha . t . E = (24 \times 10^{-6}) \times 20 \times (80 \times 10^3) \text{ N/mm}^2$$

$$= 38.4 \text{ N/mm}^2 = 38.4 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 5.2. *A brass rod 2 m long is fixed at both its ends. If the thermal stress is not to exceed 76.5 MPa, calculate the temperature through which the rod should be heated. Take the values of α and E as 17 × 10^{-6}/K and 90 GPa respectively.*

SOLUTION. Given : * Length (l) = 2 m ; Maximum thermal stress (σ_{max}) = 76.5 MPa = 76.5 N/mm^2 ; $\alpha = 17 \times 10^{-6}$/K and E = 90 GPa = 90×10^3 N/mm^2.

Let t = Temperature through which the rod should be heated in K.

We know that maximum stress in the rod (σ_{max}),

$$76.5 = \alpha.t.E = (17 \times 10^{-6}) \times t \times (90 \times 10^3) = 1.53\, t$$

$$\therefore \quad t = \frac{76.5}{1.53} = 50 \text{ K} \qquad \textbf{Ans.}$$

EXAMPLE 5.3. *Two parallel walls 6 m apart are stayed together by a steel rod 25 mm diameter passing through metal plates and nuts at each end. The nuts are tightened home, when the rod is at a temperature of 100°C. Determine the stress in the rod, when the temperature falls down to 60°C, if*

(*a*) *the ends do not yield, and*

(*b*) *the ends yield by 1 mm*

Take $E = 200$ *GPa and* $\alpha = 12 \times 10^{-6}/°C$

SOLUTION. Given : Length (l) = 6 m = 6×10^3 mm ; ** Diameter (d) = 25 mm ; Decrease in temperature (t) = 100° – 60° = 40°C ; Amount of yield in ends (Δ) = 1 mm ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm² and coefficient of linear expansion (α) = $12 \times 10^{-6}/°C$.

(a) Stress in the rod when the ends do not yield

We know that stress in the rod when the ends do not yield,

$$\sigma_1 = \alpha.t.E = (12 \times 10^{-6}) \times 40 \times (200 \times 10^3) \text{ N/mm}^2$$

$$= 96 \text{ N/mm}^2 = 96 \text{ MPa} \quad \textbf{Ans.}$$

(b) Stress in the rod when the ends yield by 1 mm

We also know that stress in the rod when the ends yield,

$$\sigma_2 = \left[\alpha t - \frac{\Delta}{l}\right] E = \left[(12 \times 10^{-6})\, 40 - \frac{1}{6 \times 10^3}\right] 200 \times 10^3 \text{ N/mm}^2$$

$$= 62.6 \text{ N/mm}^2 = 62.6 \text{ MPa} \quad \textbf{Ans.}$$